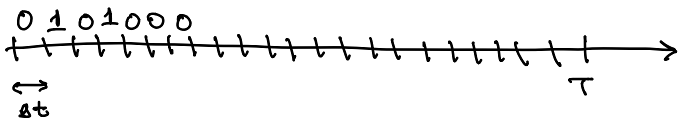
Computing Fano factor for a Poisson process



$x_1 \quad \ldots\ldots \quad x_{T/\Delta t}$

$$\begin{cases} pr(x_j = 1) = r\Delta t \\ pr(x_j = 0) = (1 - r\Delta t) \end{cases}$$

$$\langle x_j \rangle = 1 \cdot r\Delta t + 0 \cdot (1 - r\Delta t) = r\Delta t$$

$$\begin{aligned} Var(x_j) = \langle (x_j - \langle x_j \rangle)^2 \rangle &= (1 - r\Delta t)^2 \cdot r\Delta t + (r\Delta t)^2 (1 - r\Delta t) \\ &= (1 - 2r\Delta t + (r\Delta t)^2) \cdot r\Delta t + (r\Delta t)^2 - (r\Delta t)^3 \\ &= r\Delta t - (r\Delta t)^2 \approx r\Delta t \quad \text{, small } \Delta t \end{aligned}$$

spike count $\quad S = \sum_{j=1}^{T/\Delta t} x_j$

$$\langle S \rangle = \left\langle \sum_{j=1}^{T/\Delta t} x_j \right\rangle = \sum_{j=1}^{T/\Delta t} \langle x_j \rangle = \sum_{j=1}^{T/\Delta t} r \cdot \Delta t = rT$$

Recall from reading:
- x_1 and x_2 are indep. if $P(x_1 | x_2) = p(x_1)$
- point: x_i, x_j are indep $\forall\, i \neq j$ for Poisson process.

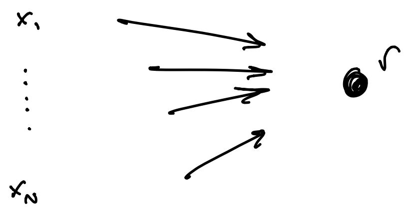
- FACT: $var(x_i + x_j) = var(x_i) + var(x_j)$, if x_i, x_j indep.

$$\rightarrow \quad \mathrm{Var}(S) = \sum_{j=1}^{T/\Delta t} \mathrm{var}(x_j) = \frac{T}{\Delta t}\left(r\Delta t - (r\Delta t)^2\right) = rT - r^2 T \Delta t$$

$$\text{FANO FACTOR} = F_T = \frac{\text{Var}}{\text{Mean}} = \frac{rT - r^2 T \Delta t}{rT} \approx \underline{1} \quad \text{as } \Delta t \to 0$$

BALANCED regime of firing in the cortex.

· Q: How is it that neurons PRODUCE this poisson firing?

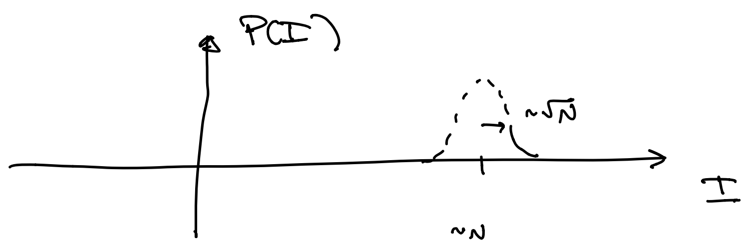


· Total input to a neuron

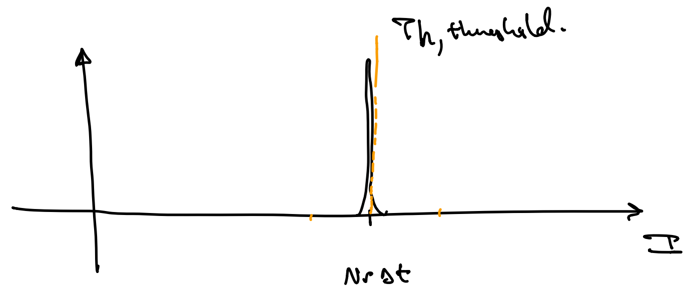
$$I = \sum_{j=1}^{N} x_j \quad , \; N \simeq 10^3$$

$$\langle I \rangle = N r \Delta t$$

$$\mathrm{var}(I) = N r \Delta t \quad ; \quad \mathrm{std}(I) = \sqrt{N r \Delta t}$$



large N limit

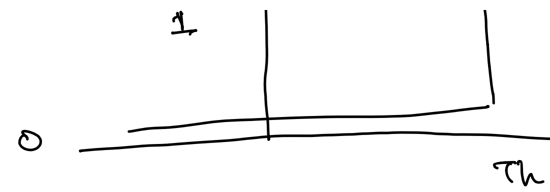


Worlds simplest spiking neuron model:

spike ($r=1$) if $I > T$

no spike if $I < T$

POINT:



only extremely finely tuned neuron, on scale of mean inputs, would produce noisy spiking w/ prob. not 0 or 1.

· Solution: Positive AND negative inputs.

$$i = 1 \ldots N/2 \quad \left[\begin{array}{ll} x_i = 1 & \text{w/ } r\Delta t \\ \quad\ \ 0 & \text{w/ } 1 - r\Delta t \end{array} \right.$$

Excitatory inputs.

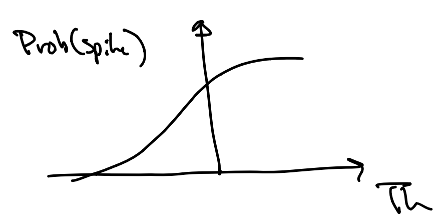
$$i = \frac{N}{2}+1 \ldots N \quad \left[\begin{array}{l} x_i = -1 \\ \quad\ = 0 \end{array} \right.$$

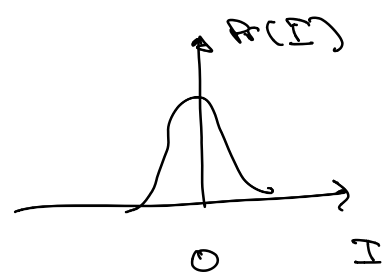
Inhibitory inputs

Then:

$$\langle I \rangle = 0$$

$$\mathrm{std}(I) = \sqrt{N r \Delta t}$$





→ No fine tuning needed for graded output firing.

Softky + Koch 1993

Shadlen + Newsome 1998

Chaotic Dynamics: van Vreeswijk + Sompolinsky, 1996.

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Final calculation ... For shadlen paper.

Say single neuron responses = $r_i + \varepsilon r_c$, common part.

Sum over N cells.

$$\sum_i (r_i + r_c) = \sum_{i=1}^{N} r_i + N \varepsilon r_c$$

$$\mathrm{Var}\left(\sum_{i=1}^{N} r_i\right) = N\,\mathrm{var}(r_i)$$

$$\mathrm{Var}(N \varepsilon r_c) = N^2 \varepsilon^2\,\mathrm{var}(r_c)$$

Fraction of var from common signal = $\dfrac{N^2 \varepsilon^2\,\mathrm{var}(r_c)}{N^2 \varepsilon^2\,\mathrm{var}(r_c) + N\mathrm{var}(r_i)} \xrightarrow{N \to \infty} 1$