Opusculum
The Euler Society Newsletter
Summer 2012 Volume 4, Issue 2

A Short Tour of the Finow Canal

At right, you can see three pictures from the Ragöser Schleuse (Ragöser Lock) of the Finow Canal. The canal was built in the years 1605-1620 on the order of Joachim III Frederick (Elector of Brandenburg), though it was largely destroyed during the Thirty Years' War (1618-1648). In 1743, construction began again (this time on the order of Joachim Frederick's descendant, Frederick II of Prussia). This incarnation of the Finow Canal was completed in 1746, and connected the Oder and Havel rivers using 15 locks [2]. In the early 20th century, the Finow Canal was superseded by the more direct Havel-Oder Canal, which permitted larger ships to be used. Since that time, the Finow Canal has been used as a recreational waterway.

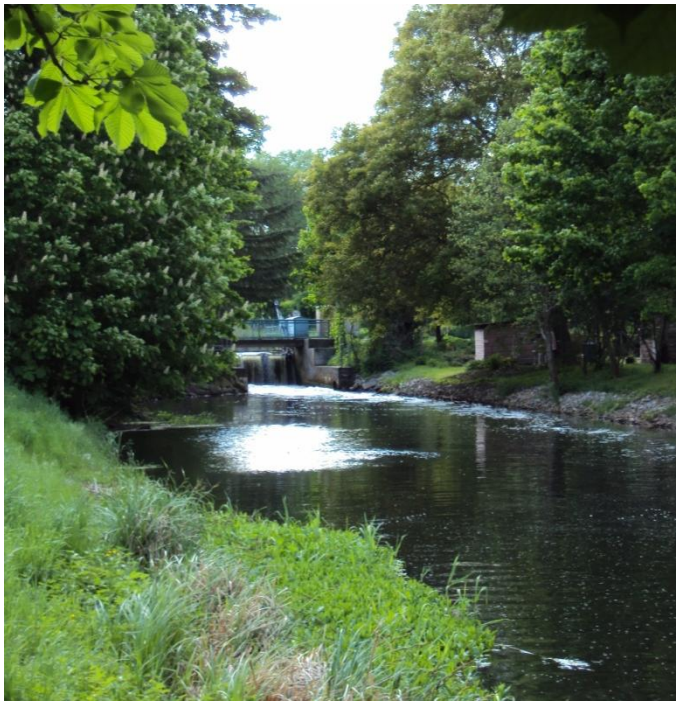
Euler's connection to the canal comes in 1749, when Frederick II tasked him with finding methods to correct the level of the canal [1]. Euler and his son, Johann Albrecht, toured the waterway in that year. In the end, their report proposed a method to regulate pressure at each of the locks. Later that year, Frederick gave Euler the job of increasing the hydraulic pressure for the fountains at Sans Souci, Frederick's summer palace.

— E. Tou

Images Courtesy Rüdiger Thiele

[1] R. Bradley, C. E. Sandifer, eds.: 2007. *Leonhard Euler: Life, Work, and Legacy*. Elsevier Science, p. 35.

[2] D. E. May, 2000: Toward a New Golden Age of Waterways? *The millennium link: the rehabilitation of the Forth and Clyde and Union Canals.* Proc. of the international conference organized by the Institution of Civil Engineers, Edinburgh, UK, p. 159-173.



Modern Corollaries to Euler

By Erik Tou

Given the sheer breadth and the foundational nature of Euler's works, it is inevitable that certain pieces—be they theorems, or notations, or equations—continue to hold relevance in a wide range of mathematical and scientific research today. Often, I find Euler's original insights are applied in a field that has progressed light-years beyond what Euler could have considered.

Tou, Continued on Page 47

Tou, Continued from Page 46

Here, I offer some of these applications, ripped from today's headlines. Often the connection to Euler is in name only, but they all illustrate the cumulative nature of scientific inquiry.

1. Buckling. From Phys.org, we have a story on the "buckliball", a roughly spherical rubber object designed to collapse when air is removed from its center.

http://phys.org/news/2012-03-buckliball-avenue-foldable.html

2. Number Theory. *Nature* reports on a new result from Terence Tao related to the Goldbach Conjecture—Tao has shown that any odd number can be written as the sum of (at most) five primes.

http://www.nature.com/news/mathematicians-come-closer-to-solving-goldbach-s-weak-conjecture-1.10636

3. Graph Theory. New research on Alzheimer's disease is combining ideas from graph theory to explore the connectivity of the brain, reports *The Guardian*. Researchers have found that Alzheimer's patients experience a degradation of connectivity in their brains.

http://www.guardian.co.uk/science/blog/2012/jul/18/alzheimers-disease-bridges-konigsberg-twitter

The Euler Line

2011 in Review

Guest Reviewer: Dominic Klyve

I wanted to take some space in this edition of *Opusculum* to mention seven notable articles relating to the work of Euler from 2011.

1. Even the most dedicated Euler Scholar could be forgiven for missing one of the more notable articles about Euler from 2011: **Colin Adams**' "Leonhard Euler and the Seven Bridges of Königsberg", in *The Mathematical Intelligencer*, Volume 33, Number 4 (2011), 18-20. The article begins:

> Once upon a time, a small boy was born in the town of Basel, Switzerland. His parents, impressed by the intelligent look in his eyes, named him Leonhard Euler, after his great uncle Leonhard, who had been smart enough to marry a countess, and now lived in a castle, giving him the right to treat his relatives like dirt....

The editor reminds us at the end of the article that "The ratio of fact to fiction in this story is precariously low", but it's a great read. Highly recommended!

2. On a considerably more serious topic is **D. D. Spalt**'s "Welche Funktionsbegriffe gab Leonhard Euler?", mentioned in the Autumn 2011 issue of this newsletter.

Here Spalt considers two different definitions that Euler gave of a function—one algebraic, denoted *fx*, and one geometric, denoted *f:x*. There are arguments here I don't recall seeing elsewhere, and no one who wishes to consider Euler's treatment of functions should ignore the text. Published in *Historia Mathematica*, v38 n4 (2011): 485-505, and available at Zentralblatt MATH 1202.01059.

3. I always like to see Euler mentioned in unusual places, so I was pleased by **Michael W. Davidson**'s recent article about Euler in—of all places—*Microscopy Today*. Although the article contains no original research, it is a nice treatment of some of Euler's work in optics. It also marks the first article I've ever purchased online—the journal charges only $1.00 per article!

The full reference: Michael W. Davidson, "Pioneers in Optics: Leonhard Euler and Étienne-Louis Malus", *Microscopy Today* (2011), 19: pp 52-54.

I would also mention, without review:

4. Andreas Verdun, "Die (Wieder-)Entdeckung von Eulers Mondtafeln." *NTM International Journal of History and Ethics of Natural Sciences, Technology and Medicine*, 19, no. 3 (2011): 271-297.

5. Weng Kin Ho, Foo Him Ho, Tuo Yeong Lee. "How Euler could have used Information & Communication Technology." Technical Report M2011-3, October 2011. *Mathematics and Mathematics Education*, Singapore.

6. G. Ferraro. "Pure and Mixed Mathematics in the Work of Leonhard Euler" in *Computational Mathematics: Theory, Methods and Applications*, Nova Science Publishers, 2011.

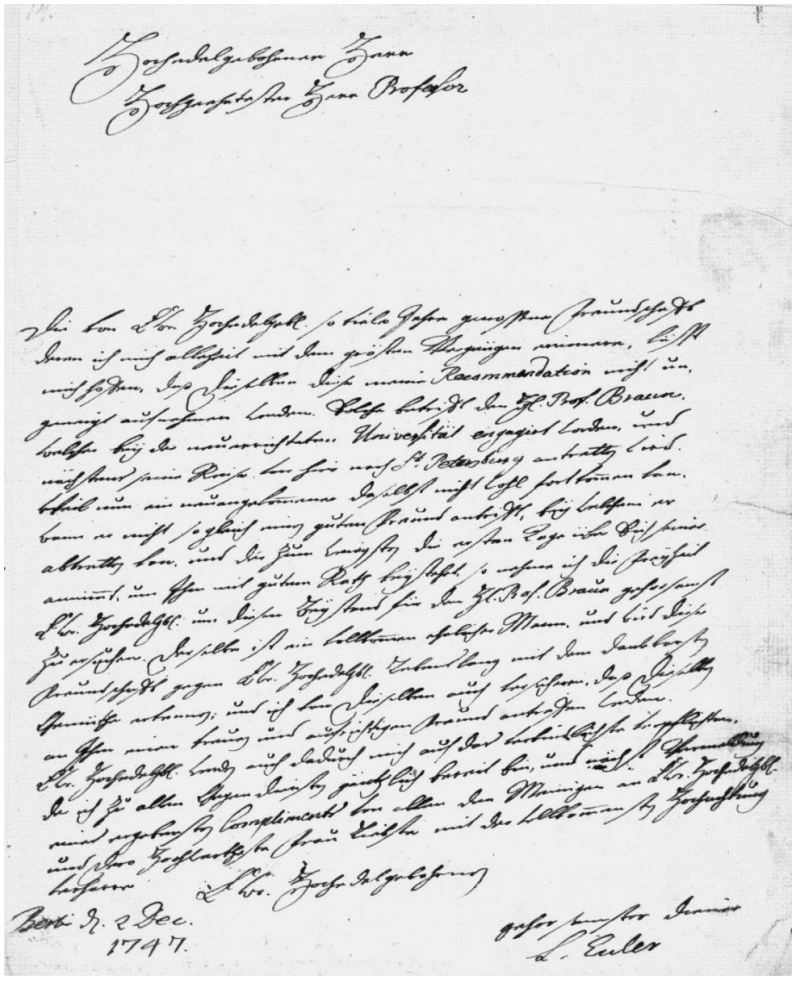
7. Marcus O. Weber and Andrea Ehrmann, "Necessary modification of the Euler–Eytelwein formula for knitting machines." *Journal of the Textile Institute*, Volume 103, Issue 6, 2012.

Euler's Home, today. The site of Euler's former Berlin home, at 21 Behrenstrasse, is now adjacent to a construction site (formerly a parking garage). The current building houses the Bavarian representative mission in Berlin. **Image courtesy Dieter Suissky.**

A New Euler Letter

By Andreas Kleinert

A letter by Euler to Johann Daniel Schumacher of November 21 / December 2, 1747, was published by A. P. Juškevic and E. Winter in *Die Berliner und die Petersburger Akademie der Wissenschaften im Briefwechsel Leonhard Eulers*, vol. 2, Berlin 1961, p. 108-110 (Nr. 2152 in vol. IVA/1 of the Euler edition, the catalog of Euler's letter list). To this letter, Euler has added another letter addressed to Christian Nikolaus Winsheim (1694-1751) in which Winsheim is asked to take care of Professor Joseph Adam Braun (1712-1768) who was expected to arrive in St. Petersburg in a near future. Euler is asking Schumacher to forward the letter to Winsheim. In a footnote (p. 109, note 4) the editors say that this letter by Euler to Winsheim could not be found ("ließ sich nicht ermitteln").



While working in the manuscript department of the University Library of Leipzig, I came across this letter that was unknown to the editors of the Euler-Schumacher correspondence and that is not listed in the catalog of Euler's letter list. The addressee's name is not mentioned, but a comparison with the Euler-Schumacher letter mentioned above makes clear that the letter must be addressed to Winsheim. The exact location of the letter is: Universitätsbibliothek Leipzig, Sondersammlungen, Slg. Neubauer, Wissenschaftler, W164.

With the permission of the manuscript department of the University Library of Leipzig, I give a transcript of the letter. All Euler scholars are invited to give comments on its reliability. A full-sized copy of the original has been posted at

http://www.physik.uni-halle.de/Fachgruppen/history/euler_winsheim.jpg.

Transription of the letter:

Hochedelgebohrner Herr
Hochgeehrtester Herr Professor

Die von Ewr. Hochedelgeb. so viele Jahre genossene Freundschafft deren ich mich allezeit mit dem grösten Vergnügen erinnere, lässt mich hoffen, daß Dieselben diese meine Recommandation nicht ungeneigt aufnehmen werden. Solche betrifft den H. Prof. Braun, welcher beÿ der neuerrichteten Universität engagirt worden, und nächstens seine Reise von hier nach St. Petersburg antretten wird. Weil nun ein neuangekommener daselbst nicht wohl fortkommen kan, wann er nicht so gleich einen guten Freund antrifft, beÿ welchem er abtretten kan, und der zum wenigsten die ersten Tage über Sich seiner annimmt, um [read: und] Ihm mit gutem Rath beÿstehet, so nehme ich die Freÿheit Ewr. Hochedelgb. um diesen Beÿstand für den H. Prof. Braun gehorsamst zu ersuchen. Derselbe ist ein vollkommen ehrlicher Mann, und wird diese Freundschafft gegen Ewr. Hochedelgb. Lebens lang mit dem dankbarsten Gemüthe erkennen; und ich kan Dieselben auch versichern, daß Dieselben an Ihm einen treuen und aufrichtigen Freund antreffen werden. Ewr. Hochedelgb. werden auch dadurch mich auf das verbindlichste verpflichten, der ich zu allen Gegendiensten gäntzlich bereit bin, und nächst Vermeldung eines ergebensten Compliments von allen den Meinigen an Ewr. Hochedelgb. und Dero Hochwertheste Frau Liebste mit der vollkommensten Hochachtung verharre

Ewr. Hochedelgebohren gehorsamster Diener

L. Euler
Berlin d. 2 Dec. 1747.

Archive and Translation Update

Alexander Aycock and **Artur Diener** continue their rapid pace of German translations, with the following works added to the Euler Archive since the last issue of *Opusculum*:

- "De valore formulae integralis $\int ((x^{a-1}\,dx)/lx)((1-x^b)(1-x^c))/(1-x^n)$ a termino $x = 0$ usque ad $x = 1$ extensae" ("On the value of the integral formula $\int ((x^{a-1}\,dx)/lx)((1-x^b)(1-x^c))/(1-x^n)$ bounded at $x = 0$ and extended to $x = 1$") [E500]
- "De numero memorabili in summatione progressionis harmonicae naturalis occurrente" [E583]
- "De summatione serierum in hac forma contentarum $a/1 + a^2/4 + a^3/9 + a^4/16 + a^5/25 + a^6/36$ + etc." ("On the summation of series contained in the form $a/1 + a^2/4 + a^3/9 + a^4/16 + a^5/25 + a^6/36$ + etc.") [E736]
- "De serie maxime memorabili, qua potestas binomialis quaecunque exprimi potest" [E743]
- "Observationes de comparatione arcuum curvarum irrectificibilium" ("Observations on the comparison of arcs of irrectifiable curves") [E252]
- "De summis serierum numeros Bernoullianos involventium" ("On the sum of series involving the Bernoulli numbers") [E393]
- "Nova methodus innumerabiles aequationes differentiales secundi gradus reducendi ad aequationes differentiales primi gradus" ("A new method of reducing innumerable differential equations of the second degree to differential equations of the first degree") [E10]
- "Exercitationes analyticae" ("Analytical exercises") [E432]
- "De constructione aequationum ope motus tractorii aliisque ad methodum tangentium inversam pertinentibus" ("On the construction of equations using dragged motion, and of other things pertinent to the inverse method of tangents") [E51]

Jacques Gélinas has completed a draft translation of "Methodus facilis inveniendi integrali huius formulae $\int (dx/x)(x^{n+p} - 2x^n \cos\zeta + x^{n-p})/(x^{2n} - 2x^n \cos\theta + 1)$ casu quo post integrationem ponitur vel $x = 1$ vel $x = \infty$ " ("An easy method for finding the integral of the formula $\int (dx/x)(x^{n+p} - 2x^n \cos\zeta + x^{n-p})/(x^{2n} - 2x^n \cos\theta + 1)$ in the case in which after integration it is put from $x = 1$ to $x = \infty$") [E620], which he has made available on Arxiv.org: http://arxiv.org/abs/1111.5571.

J. R. Stockton has completed a draft translation (with English and Latin in parallel) of "De motu rectilineo trium corporum se mutuo attrahentium" ("On the rectilinear motion of three bodies mutually attracted to each other") [E327], which he has made available on his web site:

http://www.merlyn.demon.co.uk/euler327.htm

His page also includes related commentary and research on the topic of Lagrange points.

Opusculum is the official newsletter of the Euler Society. It appears each March, July, and November.

The mission of The Euler Society is threefold: It encourages scholarly contributions examining the life, research, and influence of Euler. The Society also explores current studies in the mathematical sciences that build upon his thought. And it promotes English translations of selections from his writings, including correspondence and notebooks.

The title image for this issue of *Opusculum* is a portion of the Greater Antilles taken from the 1760 edition of the *Atlas Geographicus*, the compilation of which was supervised by Leonhard Euler. It is numbered as E205 in Enestrom's Index, and is available on the Euler Archive.