Opusculum

The Euler Society Newsletter

Autumn 2010 Volume 2, Issue 2

The Euler Society 2011 Meeting

The Euler Society Executive Committee is pleased to announce that the 2011 Meeting will be at Carthage College in Kenosha, Wisconsin.

Located between the cities of Milwaukee and Chicago, Carthage abuts Lake Michigan, at the northern edge of the city of Kenosha. There is easy access from Chicago's O'Hare airport and Milwaukee's General Mitchell airport.

The conference will run from Monday, July 24 to Wednesday, July 27. Dormitory rooms will be available for the duration of the conference, beginning on Sunday, July 23. Details about registration and membership costs, contact information, and abstract submission will follow in the Spring 2011 issue. For now, save the date!

"Bradley's Brewery" offered up a smooth porter for the Euler Society's 2010 meeting. More highlights can be found beginning on page 18.

The Omnipresent Savant

By Dominic Klyve

Euler as Master Teacher in *Letters to a German Princess*

> [*The letters to a German Princess*] *may be justly characterized as the most popular work that ever was written, and as the production of the profoundest philosopher that ever wrote*.
>
> —David Brewster, from the preface to the third English-language edition of the *Letters.*

Suppose you had a goal of writing an introduction to a short work that teaches the reader about almost everything—history and physics, astronomy and optics, logic and music, electricity and magnetism, theology and philosophy. How would you begin?

Euler's *Lettres à une Princesse d'Allemagne* sets out to do precisely this. Over the course of 234 letters written originally to Princess Charlotte Ludovica Luisa (a second cousin to Frederick the Great), and later published as a three-volume book, Euler lays out for the non-scientist the basics of a dozen disciplines. The work was immensely popular. By the end of the eighteenth century it had been translated into almost every European language, and had gone through several dozen printings.

The letters are rich with ideas, and reading them is highly rewarding. In this issue's column, we will walk through the

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New Newsletter Appearance

Regular readers of this newsletter will immediately notice a few changes in the current issue. First, and most obvious, is the title. In accordance with suggestions from Drs. Langton and Mattmüller, we have changed the title from *Opusculus* to *Opusculum*. As scholars of Latin will be aware, the standard conventions of Latin grammar dictate that the diminutive of *opus* should match the gender of its primitive: thus, the proper title is *Opusculum* rather than *Opusculus*. Secondly, some stylistic changes have been made (and will continue to be made) as we seek out a more aesthetically-pleasing format.

— E. Tou, Editor

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first letter. We will use this to study both Euler's understanding of the world, and to gain insight into Euler as a teacher.

Let's consider what things someone has to do in order to be a good teacher. While acknowledging that this question is far too complex to answer briefly, I'd like to suggest three good practices for almost any teacher:

1. Capture and maintain the students' interest.
2. Before discussing difficult concepts, introduce simpler, related ideas.
3. Leave out unnecessary details, even at the cost of perfect accuracy—especially at the elementary level.

As we will see, judged by these standards Euler's first letter is exemplary—fun to read, full of fascinating insight, and never overwhelming.

Euler decided to start his letters by titling the first with a less-than-scintillating name: *Of Magnitude, or Extension*. Of course, he is writing to a princess, and before he can do anything else, he must acknowledge her station, which he does with surprising brevity.

> The hope of having the honor to communicate in person to your highness my lessons in geometry becoming more and more distant, which is a very sensible mortification to me, I feel myself impelled to supply person instruction by writing, as far as the nature of the subjects will permit.

Having taken care of social niceties, Euler is ready to begin his science lesson. He begins with an amazingly basic concept (size) while simultaneously trying to excite the reader, promising in the first sentence to produce "examples of the smallest as well as the greatest extensions of matter actually discoverable in the system of the universe." He then does what any good teacher would do, starting with the known, and moving to the unknown. He reminds the princess that she has a good idea of what is meant by a foot, and then he exploits this understanding.

> For having the idea of a foot, we have that also of its *half*, of its *quarter*, of its *twelfth part*, denominated an inch, of its *hundredth*, and of its *thousandth* part, which is so small as almost to escape sight.

I love this sequence. Euler moves from the easily comprehensible (half a foot) to the hard-to-visualize in five steps. (A thousandth of a foot is hard to think

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The Euler Line

Special Edition: the 2010 Euler Society meeting

This year's Euler Society meeting was held at Adelphi University from July 19-21. As always, there were many interesting and engaging talks, with a wide variety of intellectual material. Topics ranged from metaphysics to history to philosophy, and of course, to mathematics. Here are a few highlights from the conference.

1. Larry D'Antonio of Ramapo College led off Day 1 of the conference with "Mating Griffins with Horses: Euler and Kant," a talk on the possible influence of Euler's Berlin circle (the Tischgesellschaft) on **Immanuel Kant's** philosophical work. Some attention was also given to the preformationism theory of human development, a now-defunct biological theory rejected by both Euler and Kant. Also of interest was the fact that Kant once wrote Euler, though he received no currently-documented reply.

2. Tom Osler of Rowan University spoke on the topic of oblique-angle diameters. This topic from analytic geometry appears in Euler's paper, "*On some properties of conic sections that are shared with infinitely many other curved lines*" [E83], which **Edward Greve** had translated from French under Osler's direction. In this paper, Euler begins with a simple definition and takes it in many unexpected directions, making use of both infinite series and differential equations.

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Front (L to R): Paul Bialek, Kim Plofker, Euler Lecturer Dave Richeson, Erik Tou, Tom Osler. Back (L to R): Bruce Burdick, Rob Bradley, Ken Gittelson, Michael Saclolo, Lee Stemkoski, Sandro Caparrini, John Snygg, Larry D'Antonio, Ron Calinger, Ed Sandifer, Bruce Petrie, Joe Malkevitch, Jordan Bell, Stacy Langton.

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about, although modern readers may have an easier time thinking of this as a third of a millimeter.) Exactly halfway through this sequence, he pauses to remind the reader that a twelfth part of a foot is a familiar unit, and then he resumes his progress toward the small.

> But it is to be remarked that there are animals, not of greater extension than this last subdivision of a foot, which, however, are composed of members through which the blood circulates, and which again contain other animals, as diminutive compared to them, as they are compared to us.

Here we see the "gee-whiz" factor that is an important part of any science writing. The existence of "animalcules" was well known by the time of Euler (Leeuwenhoek had discovered bacteria one ten-thousandth of an inch long 1676), and indeed it was known that many tiny animals (like the flea) had smaller animals living on it. Euler, however, carries this idea to its logical limit in his next passage:

> Hence it may be concluded that animals exist, whose smallness eludes the imagination; and that these again are divisible into parts inconceivably smaller. Thus, for example, though the ten thousandth part of a foot be too small for sight, and, compared to us, ceases to be an object of sense, it nevertheless surpasses in magnitude certain complete animals; and must, to one of those animals, were it endowed with the power of perception, appear extremely great.

Is Euler suggesting some sort of infinite biological regress? That all animals contain other smaller animals, all the way down? The phrase "eludes the imagination" suggests that he is, but the rest of the passage makes me think he's not – I'd love to hear from readers who have any theories about this. But back to the letter…

Euler then transitions from the small to large, and once again shines as a teacher. Using the foot as a starting point, he reminds the reader that there are 24,000 feet in a mile (this was a German mile, equal to about 7.4 km), and that miles are often easier units to deal with than feet. In so doing Euler conveys the value of wisely choosing units more simply and elegantly than do many modern science texts.

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The Euler Line [continued]

3. This year's Euler Lecture was given by **Dave Richeson**, author of *Euler's Gem*, on the topic of "Euler's Polyhedron Formula and the Euler Characteristic." Richeson detailed the history of Euler's polyhedron formula, tracking the various proof methods that were used and examining the consequent birth of the field of topology. Several interesting mathematical "gems" were discussed, including this peculiar fact:

> Given any polyhedron made up entirely of hexagons and pentagons, for which each vertex has degree three: there will always be exactly 12 pentagons.

Richeson's book was reviewed by **Jeremy Martin** in the December 2010 issue of the Notices of the AMS.

4. Bruce Petrie, of the Institute for the History and Philosophy of Science and Technology (IHPST) at the University of Toronto, spoke on "Leonhard Euler's Use and Understanding of Mathematical Transcendence." Some items of interest were:

- Euler never used the term "transcendental number," but did use "transcendental quantity."
- Euler did use the word "transcendental" to describe functions. His use of "transcendental" generally means "non-algebraic." However, he doesn't exactly explain what he means by "algebraic."
- Euler also used the terms "highly transcendental" and "interscendental."

5. Jordan Bell, **Paul Bialek**, and **Michael Saclolo** all spoke on Euler's number theory. Each of them has recently translated a work of Euler: E432 (a paper on series, in which Euler explores what would come to be known as the Riemann zeta function), E228 (Euler develops a primality test for numbers expressible as the sum of two squares), and E696/E755 (Euler considers when $x^4 + mx^2y^2 + x^4$ can be reduced to a square), respectively.

6. Ronald Calinger of The Catholic University of America gave the concluding talk of the conference by speaking on "Euler: the Final Berlin Years, 1756–1766." Calinger detailed Euler's involvement with the St. Petersburg Academy, the impact of the Seven Years' War, and Euler's stormy relationship with King **Frederick II** of Prussia. Many other interesting items were also discussed, including:

- Euler's conflict with English optician John Dollond over the achromatic telescope [partly documented in E210],
- The plundering of Euler's Charlottenburg estate,
- The reasons Euler penned his *Letters to a German Princess*,
- Euler's relations with **d'Alembert** and **Lagrange**, and
- Euler's connection to the longitude problem.

Abstracts for all speakers from the 2010 meeting can be found on the Euler Society's web site: **http://www.eulersociety.org**.

Translations of Euler's Works

As detailed in Dominic Klyve's column from Volume 2, Issue 1 of this newsletter, and reiterated by Erik Tou at this year's Euler Society meeting, the translation of Euler's works is reaching the point where a complete English rendering is within sight.

To assist in this process, the Euler Archive has created a new translation page on its web site, with two goals in mind:

1. Inform the community of Euler scholars on current and recent translation efforts,
2. Prevent duplication of translations.

While the present page documents only the English translations of Euler's works, the Euler Archive is working toward a comprehensive documentation of *all* translations. The direct URL is:

http://www.math.dartmouth.edu/~euler/translations.html

The page can also be found by going to the Euler Archive main page (http://www.eulerarchive.org) and selecting "Translations" from the navigation bar.

On this page, you will find a listing of many current translation projects, in addition to a listing of completed translations. At present, the Euler Archive has documented 155 complete English translations of Euler's works.

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> …saying that the distance from Berlin to Magdeburg is 18 miles is much clearer than if the distance between these cities was said to be 432,000 feet: a number so great almost overwhelms the understanding.

Yet another thing a good teacher must do is refrain from overwhelming his students with unnecessary detail, even if it means bending the truth. Euler clearly knows this, as he writes

> Again, we have a tolerably just idea of the magnitude of the earth, when we are told that this circumference is about 5400 miles. And the diameter being a straight line passing through the center, and terminating in opposite directions, in the surface of the sphere, which is the acknowledged figure of the Earth, for which reason also we give it the name of *globe* – the diameter of this globe is estimated to be 1720 miles.

Of course, the Earth is not shaped like a sphere, and Euler knew this well. Maupertuis, La Condamine, and Bouguer (among others) had made famous voyages to measure a degree of longitude in Lapland and South America in the 1730s. Euler was among those who used their measurements to calculate that the Earth was wider at the equator than at the poles, confirming a prediction of Newtonian physics [E32]. This story is fascinating, but Euler resists the temptation to tell it, sticking instead to a gentle fiction which furthers his teaching goals in this letter.

Once he establishes the diameter of the earth as a unit, Euler proceeds to discuss the distance to the moon and the sun, demonstrating again the value of choosing the right unit:

> Of all the heavenly bodies the moon is nearest us, being distant only about 30 diameters of the earth, which amounts to 51,600 miles, or 1,238,400,000 feet; but the first computation of 30 diameters of the earth is the clearest idea. The Sun is about 400 times farther from us than the moon, and when we say its distance is 12,000 diameters of the earth, we have a much clearer idea than if it were expressed in miles or feet.

Euler moves next to the rest of the solar system, giving the distance to the sun, and also naming the planets. This section is particularly interesting in the most commonly seen translation, in which Euler is supposed to have written

> Besides the *Earth*, there are ten other similar bodies, named planets, which revolve round the sun; two of them at smaller distances, *Mercury* and *Venus*; and eight at greater distances, namely, *Mars*, *Ceres*, *Pallas*, *Juno*, *Vesta*, *Jupiter*, *Saturn*, and the *Georgium Sidus*.

I have seen this list mentioned in many places; student reports and slides of talks given at professional conferences are available online which mention this list, largely because it's so fascinating. The careful reader should be wondering, however, just how Euler predicted these "planets", many of which weren't discovered until half a century after this letter was written. The short answer, of course, is that he didn't. The long answer is more interesting. Explaining what Ceres *et al.* are doing in this translation requires understanding why the *Letters to a German Princess* were translated in the first place. This question will be the subject of my next column. For now, let's return to Euler's letter.

Beginning with a foot, Euler has moved up in size to distances between cities, and then to the moon, and sun, and the planets. He next turns to the stars. Of particular note in the following passage is just how ignorant eighteenth-century astronomers were about the size of the universe.

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Dinner at the Lucky Duck in Garden City, NY. In the foreground (clockwise from right) are: Stacy Langton, Paul Bialek, and Jordan Bell.

Solution: A New Twist on an Old Problem

D 2	A 3	C 4	E 1	B 5
E 5	C 2	D 1	B 3	A 4
B 4	D 5	E 3	A 2	C 1
C 3	B 1	A 5	D 4	E 2
A 1	E 4	B 2	C 5	D 3

This is the solution to the Graeco-Latin Sudoku puzzle, which originally appeared in Issue 1, Volume 2 of *Opusculum*. An interesting Graeco-Latin Square generator (for 3x3, 4x4, 5x5, 7x7, and 9x9 puzzles only) is available online.

It is well-known that there exist Graeco-Latin squares of any dimension $n \geq 3$, with the exception of $n = 6$. Euler himself conjectured that there could not exist a Graeco-Latin square of dimension $n \equiv 2 \pmod 4$. This was due to his inability to solve the 36-officer problem (see **E530** for more information).

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> All the other stars which we see, comets excepted, are called fixed; and their distance from us is incomparably greater than that of the sun. Their distances are undoubtedly very unequal, which is the reason that some of these bodies appear greater than others. But the nearest of them is, unquestionably, above 5000 times more distant than the sun: its distance from us, accordingly, exceeds 45,000,000 diameters of the earth, that is 77,400,000,000 miles, and this again, multiplied by 24,000, will give that prodigious distance in feet. And this, after all, is the distance only of those fixed stars which are the nearest to us – the most remote which we see are perhaps a hundred times farther off.

Recall that Euler's miles are German miles. Converting to standard English miles, we see that he believes the closest star to be more than 356 billion miles away. While true, this is a significant underestimate – the closest star to Earth, Proxima Centauri, is 70 times farther away.

Even more interesting here is the fact that Euler seems to assume that all stars are equally bright; only their distance from us makes them appear brighter or dimmer. Even assuming that the stars are equally bright, however, Euler's numbers are suspect. The brightest star in the sky, Sirius, is roughly 300 times brighter than the dimmest stars (magnitude 6 stars, in modern terminology). Since brightness falls off with the square of the distance (a fact which Euler would have understood well), it might be more reasonable to assume that the farthest stars are 90000 times farther than the closest ones – again, assuming that they are all equally bright. Instead, Euler seems to suggest that the brightest stars are only 10 times brighter than the dimmest. To be fair, accurate measurements of stellar brightness would have to wait almost a century, when Sir John Herschel and others computed just how much brighter the bright stars are than the dim ones. Nevertheless, Euler's estimate is surprisingly far off.

Earlier, Euler suggested that the smallest beings are far smaller than those which we currently know. He concludes this letter with similar thoughts in the other direction, with a final, almost perfunctory, nod toward God.

> It is probable, at the same time, that all these stars taken together constitute only a very small part of the whole universe, relative to which these prodigious distances are not greater than a grain of sand compared to the earth. This immensity is the work of the Almighty, who governs the greatest bodies and the smallest.

Euler's letter is a virtual treasure trove, full of fascinating insights into his views into the natural world and the nature of science. Through all of this, his skill as a master teacher—one who never loses sight of his student—shines through. Many others have noticed the same thing about the *Letters*, which is one of the reasons that they have been so widely published and translated. The story of exactly how that process came about will be the subject of my next column…

— D. Klyve

Translation and Archive Update

Ian Bruce continues his translations of Euler's books, this time producing an English rendering of the *Institutionum Calculi Differentialis* [E212]. While John Blanton has produced an English translation of Part I, Bruce has committed himself to producing an independent translation of the entire work. Thus far, he has translated all of Part I and Chapters 1-5 and 18 of Part II. All of these are available directly on Bruce's web site:

http://www.17centurymaths.com/contents/differentialcalculus.htm

Google Books has several of Euler's works provided both in the original language and in translation. Some items you may not have discovered are:

- **[E212]** John Blanton's English translation of Part I of the *Institutionum Calculi Differentialis* is available, with some minor abridgements:

  http://books.google.com/books?id=RqFc0GC1RhcC

- **[E387]** An 1810 English translation of Euler's *Elements of Algebra* is available:

  http://books.google.com/books?id=mqI-AAAAYAAJ

- **[E102]** Blanton's English translation of Volume 2 of the *Introductio in analysin infinitorum* has been made available:

  http://books.google.com/books?id=LcLq6tt2kFYC

- **[E426]** An English translation of Euler's shipbuilding text, *Theorie complete de la construction et de la manoeuvre des vaisseaux*, is also available:

  http://books.google.com/books?id=5ysUAAAAQAAJ

  This translation dates from 1790, and was produced by Henry Watson. It includes Euler's supplementary material, *Sur l'action des rames*, concerning the motion of oars.

[E228] Paul Bialek has recently completed an English translation of *De numeris qui sunt aggregata duorum quadratorum*. In this paper, published in 1758, Euler develops a method to test numbers for primality, depending on whether or not they are uniquely representable as the sum of two squares. This translation will appear on the Euler Archive.

[E241] Mark Snavely and **Phil Woodruff** have recently translated *Demonstratio theorematis Fermatiani omnem numerum primum formae* $4n+1$ *esse summam duorum quadratorum* into English. Published in 1760, this paper details Euler's proof of the fact that $a^{4n} - b^{4n}$ is divisible by $4n+1$.

[E561] Jordan Bell has translated *Variae observationes circa angulos in progressione geometrica progredientes* into English. This 1773 paper deals with series involving trigonometric terms, taking the double-angle identity $\sin(2x) = 2\sin(x)\cos(x)$ as the starting point.

All documents mentioned in this column are available via the Euler Archive:
http://www.eulerarchive.org.

Opusculum is the official newsletter of the Euler Society. It is published on a quarterly basis.

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Letters, articles, and other contributions to the *Opusculum* are very welcome. Send any contributions, observations, or news items to Erik Tou at etou@carthage.edu.

The mission of The Euler Society is threefold: It encourages scholarly contributions examining the life, research, and influence of Euler. The Society also explores current studies in the mathematical sciences that build upon his thought. And it promotes English translations of selections from his writings, including correspondence and notebooks.