

Did Cotes Anticipate Euler?

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Ex quibus intelligitur quomodo quantitates exponentiales imaginariæ ad Sinus & Cofinus Arcuum realium reducantur. Erit vero $e^{+v\sqrt{-1}} = \text{cof. } v + \sqrt{-1} \cdot \text{fin. } v$ & $e^{-v\sqrt{-1}} = \text{cof. } v - \sqrt{-1} \cdot \text{fin. } v$.

You don't need to know any Latin to recognize Euler's Formula as it appeared in §138 of his *Introductio in analysin infinitorum* (1748), even if it is not in the more popular form of Euler's Identity, $e^{i\pi} + 1 = 0$ (when did this first appear?).

The [Wikipedia](#) page on Roger Cotes claims that he "introduced what is known today as Euler's Formula." But is this so?

The only paper which Cotes published in his lifetime, "Logometria," was published in the *Philosophical Transactions of the Royal Society of London* in 1714. In that paper he wrote:

triangulum designari, sed modo inexplicabili. Nam si quadrantis circuli quilibet arcus, radio CE descriptus, finem habeat CX finemque complementi ad quadrantem XE : sumendo radium CE pro Modulo, arcus erit rationis inter $EX + XC\sqrt{-1}$ & CE mensura ducta in $\sqrt{-1}$. Verum isthæc aliis, quibus operæ pretium vide-

"For if some arc of a quadrant of a circle described with radius CE has sine CX and sine of the complement of the the quadrant XE , taking radius CE as modulus, the arc will be the measure of the ratio between $EX + XC\sqrt{-1}$ & CE , the measure having been multiplied by $\sqrt{-1}$."

This translation is from *Roger Cotes — Natural Philosopher* (1983), p. 170, by Ronald Gowing. He makes no mention of this passage containing Euler's Formula. That is not surprising, for there is no formula here, just a verbal statement. Can you interpret this as a statement of Euler's Formula? Wikipedia does, but it takes some effort.

Paul Nahin in his *An Imaginary Tale: The Short History of $\sqrt{-1}$* (1998, p. 166) remarks that this bit of Cotes's work "went unnoticed for 185 years, until a Russian mathematician brought it to the world's attention in an 1899 book on the history of functions." Who was this Russian mathematician and what is the title of his book on the history of functions?

The purpose of this note was to ask: What does "anticipate" mean? Did Cotes think he had Euler's Formula? Just because modern mathematicians can manipulate the text to obtain Euler's Formula, does that count as anticipation? RSVP.