Abstracts

The Euler Lecture – E228

V. Frederick Rickey, USMA

How's that for the shortest title ever? How can you decide if a number is the sum of two squares? Euler begins with the dumbest possible algorithm you can think of: Take the number, subtract a square, and check if the remainder is a square. If not, repeat, repeat, repeat. But Euler, being Euler, finds a way of converting all those subtractions into additions. Then he does several things to speed up the computation even more (but, sadly, does not explain himself very well). He applies this to 1,000,009, and — in less than a page — finds that there are two ways to express this as a sum of squares. Hence, by earlier work in E228, it is not a prime. Amusingly, when he later described how to prepare a table of primes "ad milionem et ultra" (E467), he includes this number as prime. So then feels obliged to write another paper, E699, using another refinement of his method, to show that 1,000,009 is not prime.

Euler's model for the shape of a fluid surface kept in equilibrium by the action of the wind

Sylvio Bistafa, University of Sao Paolo

From a translation of E494 "*De Figura Quam Ventus Fluido Stagnanti Inducere Valet*" (On the Figure that the Wind can Induce on a Stagnant Fluid), it will be shown how Euler models a fluid hump in the fluid surface, kept in equilibrium by the action of the wind against the fluid gravitational force. According to Euler, different hump-shapes could be formed according to how the effect of the wind angle of incidence is taken into account. Euler considers first the case where the effect of the wind is proportional to the square of the sine of the angle of incidence (a result proved earlier in *"Scientia navalis seu tractatus de construendis ac dirigendis navibus*"), showing that the resulting figure would be a cycloid. In the second case, the wind effect is modeled as being proportional to the sine of the angle of incidence, showing that the resulting figure would be a tractrix. The third case considers a composition of these two previous cases, showing that the resulting figure would be something between these two curves, according to a factor of proportionality applied to each case. A short discussion about these curves will be presented. At the end Euler recognizes that this exercise would do little in advancing the theory for the motion of fluids.

Polar Ordinates from Newton to Euler

Robert E. Bradley, Adelphi University

According to the usual narrative, priority for the invention of polar coordinates belongs to Newton, although Jakob Bernoulli has priority of publication in 1691, because Newton's results were only published posthumously in 1736. However, these early versions were not polar coordinates in the form that would be recognized by today's readers. They featured ordinates emanating from a single point or pole, with some geometric construction playing the role that today belongs to an angular coordinate. The largest and most accessible collection of these early schemes of polar ordinates is probably to be found in l'Hôpital's *Analyse des infiniment petits* (1696), based on the lessons given to the Marquis by Johann Bernoulli.

It was only in Euler's *Introductio in analysin infinitorum* (1748) that polar coordinates were first employed in their modern form. In this talk, I will describe Bernoulli's approaches to polar ordinates, as presented in l'Hôpital's textbook, and contrast these with the system introduced by Euler in the *Introductio*.

"We have no ideas of the infinitely small": Euler and Infinitesimals

Lawrence A. D'Antonio, Ramapo College

The title quote is from a late work of Newton, which severely criticized the reliance on infinitesimals in the calculus of Leibniz. It is true that Leibniz used infinitesimals but it is also clear that their role in his work is only that of useful fictions. For Leibniz, infinitesimals represent a certain mode of calculation, whereas for Euler they are woven throughout his notions of analysis. In this talk we will consider how for Euler, infinitesimals, contrary to Newton, are central to the presentation of analysis in the *Introductio* and the *Institutiones calculi differentialis*.

What did Euler learn from d'Alembert about fluid mechanics?

Stacy Langton, University of San Diego

In 1752, Euler presented his paper "*Principia Motus Fluidorum*" (E258) to the Berlin Academy. With the exception of some papers on hydraulics, this paper was Euler's first major work in fluid mechanics. (The papers E225 and E226, though published earlier, were written later.) In the paper E258, Euler wrote down, for the first time, the general equation, now called "Euler's equation" for the motion of a (non-viscous) fluid. (During the 19th century, Navier, Poisson and Stokes found the appropriate way of representing the effects of viscosity, to form the "Navier-Stokes equation".)

D'Alembert's "*Traité de dynamique*" was published in 1743, followed in 1744 by his "*Traiteé de l'équilibre et du mouvement des fluides*". In 1748, the Berlin Academy proposed a prize competition on the subject of the resistance of fluids. For this competition, d'Alembert wrote his "*Essai d'une nouvelle théorie de la resistance des fluides*". Euler was certainly one of the judges. The Academy decided that no contestant had earned the prize. D'Alembert angrily published his essay in 1752.

In this talk, I will try to explore what Euler could have learned from d'Alembert that he was able to apply in his own work on fluid mechanics. I do not promise that I will be able to give a definitive answer to the question! But I will try to explain what d'Alembert did, as well as I am able to understand it.

Euler's Imprint on the French Enlightenment through the *Encyclopédie* of Diderot and d'Alembert

Michael P. Saclolo, St. Edwards University

By the time the first volume of the *Encyclopédie, ou dictionnaire raisonné des sciences, des arts et des métiers* was published in 1751, Euler was already a well-established reputed scholar and well-known in European scholarly circles, including the Royal Academy of Sciences in Paris. It is therefore not surprising to find Euler's presence in the *Encyclopédie*, one of whose editor's is d'Alembert, with whom Euler maintained a correspondence. Euler is mentioned or cited in several articles of the *Encyclopédie*, many of them written by d'Alembert himself. A few of his works, such as the *Introductio*, the *Mechanica*, and his solution to the Seven Bridges of Königsberg problem, are cited by title. We shall look at how Euler's ideas are reflected within the entries of the *Encyclopédie*. Does the exposition of the articles paint Euler as the authority on the subject, and do his ideas either corroborate or oppose those of his contemporaries? As an attempt to answer these questions, we hope to catch another glimpse of his influence and impact on the scholarship of those times.

The method of parallel sections in the early days of fluid mechanics

Erik Tou, University of Washington, Tacoma

During their years living together in St. Petersburg, Euler participated in Daniel Bernoulli's early inquiries into the science of fluids. Bernoulli's work culminated in 1738, with the publication of *Hydrodynamica*. Shortly thereafter, Daniel's father Johann shared an early manuscript of his own with Euler, which coincidentally arrived at Euler's desk around the same time as Daniel's completed text. In these texts, and also in Euler's correspondence from this time, the method of parallel sections was employed as a first application of the Calculus to the behavior of fluids, specifically in the case of water draining from the bottom of a cistern. In this talk, we briefly compare three examples from Euler and Bernoullis *père* and *fils*.

Euler's Publication Record During the Seven Years' War

Erik Tou, University of Washington, Tacoma

The Seven Years' War (1756-1763) was arguably the first global war, with the belligerent powers fighting battles on several continents. Prussia's situation was threatened more than most, with numerous battles occurring on its soil. Euler spent these years at Berlin, which was itself raided by Austrian and Russian forces in 1760. In this talk, we explore Euler's work during these years, and do a bit of data mining to see where and how he shared his work with the world.