Euler 2014 :: Abstracts

The Euler Lecture

Alberto Martinez, University of Texas—Austin

The Elegance of Euler's *Algebra*

Abstract.
Traditionally, historians have praised Euler's *Algebra* of 1770 as one of the most widely read math books in history, second only to Euclid's *Elements*. At the same time, historians and commentators have made disparaging comments about various elementary aspects of Euler's *Algebra*, including his rules for operating with zero and with imaginary numbers. They comment that blind, old Euler made silly mistakes, that he was confused about basic operations, and that the book suffered of transcription errors or printing miscues. I will argue that such claims are mistaken. Instead, I will show the internal consistency of Euler's expressions and I will explain that his algebra actually constitutes an alternative to ordinary algebra as we know it, and that it elegantly lacks, and does not need, certain artificial rules that ordinary algebra includes in order to not yield contradictory results.

Sylvio R. Bistafa, University of São Paulo Polytechnic School

Euler’s Friction of Fluids Theory and the Estimation of Fountains Jet Heights

Abstract.
According to the records, a treatise by Euler with the title *Tentamen theoriae de frictione fluidorum* was presented to the St. Petersburg Academy on June 17, 1754, and perhaps was read to the Berlin Academy on December 2, 1751. At this time, the King of Prussia, Frederick the Great, was pretty much involved in the construction of a water park at Sanssouci, and engage Euler to improve the water-raising machinery and the tubes for the pipeline to the elevated reservoir. From this reservoir, the water would be guided to a main fountain and four smaller fountains. Although there is no evidence that Euler was directly involved with the hydraulics of the fountains, a contention is that his treatise on fluid friction had the goal of calculating the head-losses in the conduits, which would determine the fountains jet heights. In this treatise, Euler assumes that as for the solid friction, the fluid friction is proportional to the applied normal force, not realizing that fluid friction is a viscous effect, independent of the applied pressure. Several experiments are suggested by Euler to obtain a friction factor, which would then experimentally close his equations. Another contention is that Euler himself never performed neither one of the suggested experiments—he simply guessed a friction factor. A detailed development of five different problems of discharge are presented in his treatise, taking into account the

loss of head in the conduits. In the appendix, an example is given for the calculation of the jet heights of a particular fountain, fed with conduits of different amplitudes. One more contention is that this example was proposed with the main fountain at Sanssouci in mind. By applying current procedures for the calculation of head-losses in pipes, it is shown that Euler grossly overestimated the jet heights of this fountain. A broader picture emerges from a comparative analysis of conduits with different lengths and amplitudes, which shows that the head-losses are consistently underestimated by Euler's theory.

Robert E. Bradley, Adelphi University

The Pure Within the Applied: Euler in the Berlin *Mémoires*

Abstract.
During his Berlin Period (1741-1766), Leonhard Euler was a key figure in the revitalized Berlin Academy. He supported the academy's mission, both as an able administrator and as a prolific author in the pages its journal. Among his many publications in the academy's *Mémoires*, topics in applied mathematics are particularly well represented, including astronomy, optics, mechanics, fluid dynamics, and electricity & magnetism. During the same period, Euler published many articles in the St. Petersburg journals, largely concerning pure mathematics, particularly in number theory and infinitesimal analysis. In this talk, we survey the broad outlines his Euler's research agenda in Berlin, paying particular attention to the papers in pure mathematics that he chose to publish in the Berlin journal.

Lawrence D'Antonio, Ramapo College of New Jersey

Euler and the Academy Prize Competitions

Abstract.
The Paris and Berlin Academies of Science were major institutions for mathematics and natural philosophy in the 18th century. These and similar academies, such as the St. Petersburg Academy and the Royal Society of London, sponsored research, gave academy members opportunities to present their research in oral and written form, and encouraged communication among the prominent scholars of the Enlightenment. In short, the academies played a central role in the Republic of Letters. Several academies ran prize competitions in which a particular problem was posed, submissions were accepted and the winning submission(s) were awarded a prize. We focus on the prize competitions run by the Paris and Berlin Academies, both of which figured prominently in the career of Euler. We look at the history of these competitions, Euler's contributions, and examine particular competitions in detail, e.g., the controversial 1747 Berlin prize competition on the theory of monads.

Stacy Langton, University of San Diego

Kirchhoff's 1850 treatment of vibrating elastic plates

Abstract.
In the second part of his great book on the calculus of variations, the "Methodus Inveniendi" of 1744 (E65), Euler, following a suggestion from Daniel Bernoulli, derived the equations of motion of an elastic rod, or "elastica", using a variational argument. The "constitutive" assumption that Euler made about the elastic rod was that the bending moment at any point of the rod was proportional to the curvature. This assumption goes back originally to Jacob Bernoulli.

Many years later, in his E410 (1771) and E481 (1776), Euler rederived the equations of the elastica using a direct argument from the balance of linear and rotational momentum, rather than by a variational argument. Then, in E526 (1782), he used these derivations to study the small vibrations of an elastic rod, getting a fourth-order partial differential equation.

In 1809, the Paris Institute announced a prize competition for the mathematical analysis of the vibrations of an elastic plate. Sophie Germain, intending to compete for the prize, began by studying Euler's E526. In order to generalize Euler's method to vibrations of a plate, she had to find a generalization of the Bernoulli-Euler constitutive assumption. The assumption she made was that the bending moment in the elastic plate was proportional to what we now call the "mean curvature" of the surface.

Using this assumption, Germain derived a sixth-order equation for the vibrations. Lagrange then showed that Germain had made a mistake in her calculation. Starting from her assumption about the mean curvature, he derived a fourth-order equation, which in fact is the equation generally accepted today.

In 1850, Kirchhoff reconsidered the equation of the vibrating plate. He showed that Germain's assumption about the mean curvature was wrong, because it led to incorrect conclusions. He then gave his own analysis of the vibrating plate.

In this talk, I will describe this work of Kirchhoff. In particular, I will show how he disproved Germain's hypothesis, and I will discuss the constitutive assumption he used to replace it.

V. Frederick Rickey, USMA West Point

Coming Soon to a Library Near You: The NEW Euler-Goldbach Correspondence

Abstract.

While Euler and Goldbach corresponded over a period of 35 years, very few of their letters have been translated into English. This is about to change with the publication of

volume IVA4 of Euler's *Opera Omnia*, edited by Franz Lemmermeyer and Martin Mattmüller.

The biggest change from Fuss's 1843 edition of the correspondence is that now the opening and closing portions of the letters are included. They don't deal with mathematics, but they do contain a great deal of interesting information about the two men and their circle. While this information was included in the 1965 edition by Juv̌skeviv̌c and Winter, it is not well known. We shall discuss some of it. This presentation will deal primarily with the rich ideas in the correspondence and attempt to convey the excitement of mathematics being created that is so obvious when one reads through their correspondence.

Michael Saclolo, St. Edward’s University

The Rules and Regulations of the Berlin Academy at time of Euler's Tenure

Abstract.
The constitution adopted by the Royal Academy of Sciences in 1746 prescribed the rules and regulations that governed much of Euler’s tenure during his years in Berlin. In this talk we shall look at the inner workings of the revitalized Berlin Academy through the lens of these rules and regulations. They stipulate the organizational structure of the Academy, including its divisions, leadership, and the types of membership. They specify the authorities and responsibilities of its President and other members of the Directorship, as well as the academic committee that decides what works are published in the official record. They outline rules for nominations, elections and replacement of the Academy’s membership, the frequency of meetings, and even official breaks. They specify requirements of tenure including the number of times a member needs to present in front of the Academy. Finally, they refer to the annual competition that the Academy organizes for the best paper on an announced topic.

Ajay Sharma, Fundamental Physics Society

Isaac Newton, Leonhard Euler, and *F=ma*

Abstract.
In the existing literature statement “rate of change of momentum is proportional to impressed force” is used as definition for Newton’s second law. Neither this definition nor equation $F = ma$ was never given by Newton in the Principia, but credited to Newton unscientifically. On the other hand Newton defined second law in the Principia as “alteration of motion is proportional to impressed force.” Newton has defined terms inertia, motion, force, centripetal force, axioms, scholium, etc. as it was beginning of developments of physics. So Newton did not give any equation, as the equations were developed at later stage. In 1736 Swiss mathematician Euler put forth force as $F = ma/n$

(where n is constant and depends upon unity of measure). Further in 1750, Euler using extrinsic references frames (a system of three orthogonal Cartesian axes), gave mathematical equation $F = 2m\mathrm{d}^2s/\mathrm{d}t^2 = 2ma$ (the coefficient 2 depended on the unity of measure). Euler capriciously divided RHS by 2 to obtain $F = ma$, which contravenes the definition of equation. The coefficients must be same in both equations of force, i.e., 2. Euler used two primary or fundamental units L (length) and F (force), hence value of coefficient is 2. The systems of primary units L (length)–F (force)–T (Time) and L (length)–M (mass)–T (Time) were introduced in the following century. Thus in order to obtain all-encompassing mathematical form of Newton's second law ($F = ma$) arbitrariness is the basis. Now $F = ma$ is incorrectly associated with Newton in existing standard literature, and $F = ma$ is arbitrarily obtained from equation $F = 2ma$ or $F = ma/n$, by Euler. Thus $F = ma$ may be understood as a postulate.

Erik Tou, Pacific Lutheran University

D. Bernoulli, Euler, and the Development of Fluid Mechanics

Abstract.
In modern fluid mechanics, mathematical models are suffused with functions and vectors. In this analytic system, theorems and principles are derived by algebraic means, and geometry is used merely to illustrate the problem at hand. But it was not always so! Before the 19th century movement to foundations in mathematics, scientists and mathematicians derived results using infinitesimals and a heavy dose of Euclidean geometry. While less rigorous than modern methods, the geometric approach permitted researchers to derive many of the formulas that undergird the full theory developed later.

In this talk, we will examine this early, geometric approach through the lens of 18th century fluid mechanics. The story begins with the work of Daniel Bernoulli, whose book, *Hydrodynamica* (1738), is viewed as the earliest text on fluid mechanics. We will look at one particular example from Section XII, in which Bernoulli analyzes the motion of water in a pipe. Then, we will move to the 1750s and the work of Leonhard Euler. Fortunately, several of Euler's papers and manuscripts from this period are available, and provide us with a timeline for Euler's thought in this area. We will look at three of Euler's geometric arguments. The ultimate goal of this research, which we will touch on briefly, is to trace the origins of the so-called Bernoulli equation for fluid flow.
