Euler 2010 - Abstracts

Adelphi University
Garden City, NY 11530

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The Euler Lecture

David Richeson, Dickinson College

Euler's polyhedron formula and the Euler characteristic

Abstract: In 1751 Euler discovered that any polyhedron with V vertices, E edges, and F faces satisfies $V - E + F = 2$, but the proof he gave was flawed. Many rigorous proofs followed, but so did a number of "counterexamples." It took 150 years for mathematicians to fully understand this simple formula and discover the more general Euler characteristic. Today Euler's formula is held aloft as one of the most beautiful theorems in all of mathematics and the first great theorem of topology. In this talk we will give the history of Euler's theorem and give a sampling of its many applications.

Jordan Bell, University of Toronto

Euler and the zeta function

Abstract: I will discuss Euler's 1772 paper "Exercitationes analytica," E432. In this paper Euler gets an expression for the sum of the reciprocals of the cubes. The paper begins with a statement of the functional equation for the Riemann zeta function, which Euler formulates in terms of divergent series. More generally, I will give some thoughts on Euler's use of divergent series.

Paul Bialek, Trinity International University

Euler's primality test for numbers of the form $4n + 1$

Abstract: Fermat was the first to conjecture that an odd prime p can be expressed as the sum of two squares $x^2 + y^2$ if and only if p is congruent to 1 (mod 4). In his paper, 'On numbers which are the sums of two squares" [E228], Euler outlines a proof which he admits is incomplete. He does, however, prove that numbers of the form $4n + 1$ are prime if they can be expressed in exactly one way as a sum of two squares and composite if they can be expressed in more than one way as a sum of two squares. Euler recognizes that this is a primality test for numbers of the form $4n + 1$ and gives several examples. We will present a translation from the Latin and a summary of this previously untranslated paper.

Robert E. Bradley, Adelphi University

Euler's Algebraic Analysis

Abstract: From the 17th century onwards, mathematicians could expand a wide variety of functions into power series without explicit use of methods from the differential calculus, such as Taylor's Theorem. Such "precalculus" techniques did not originate with Euler, but were popularized in his *Introductio in analysin infinitorum* (1748) and were subsequently taught into the 20th century. We examine Euler's contributions to the field of Algebraic Analysis. We also consider Cauchy's approach to making such series expansions rigorous, as presented in his *Cours d'analyse*.

Bruce S. Burdick, Roger Williams University

Various Observations on Euler's E72

Abstract: Euler's *Variae observationes circa series infinitas* (E72) considers a variety of infinite sums and products. His first theorem,

$$\frac{1}{3}+\frac{1}{7}+\frac{1}{8}+\frac{1}{15}+\frac{1}{24}+\frac{1}{26}+\frac{1}{31}+\frac{1}{35}+\cdots=1,$$

where the denominators are the whole numbers that are one less than a nontrivial power, he attributes to Golbach, both for its statement and its proof. He then proceeds to prove other theorems in more or less the same manner.

The method of choice for Euler (and presumably Goldbach) involves subtracting infinite quantities from infinite quantities in a way that would no longer be acceptable as a mathematical demonstration. In a recent paper, Edward Sandifer and the speaker gave a modern proof of Euler's Theorem 1. This talk is a follow-up to that paper, and will show that other theorems from E72 can be supplied with proofs that meet the present-day standards of rigor.

Ronald Calinger, The Catholic University of America

Euler: the Final Berlin Years, 1756 - 1766

Abstract: This paper examines Euler's interactions with the St. Petersburg Academy, the impact of the Seven Years' War, Euler's relations with Lagrange, and the Berlin Academy after Maupertuis. It proceeds to the competition with Dollond over the achromatic telescope, the reasons for the Letters to a German Princess, Chess, Euler at war, and the plundering of the Charlottenburg Estate, the Euler disc, d'Alembert's visit to Berlin in 1763, the longitude prize, and Lambert. It closes with tensions with Frederick II, the complex reasons for leaving Berlin, and Rigid Bodies.

Sandro Caparrini, Institute for the History and Philosophy of Science and Technology, University of Toronto

Rigid Bodies in the Eighteenth Century

Abstract: This is a succinct but detailed discussion of the early development of the general theory of rigid bodies, mainly centered on Euler's great memoirs from the 1750s. Some important works by other mathematicians will be analyzed as

well: Johann Bernoulli's work on the "center of spontaneous rotation" (1740), d'Alembert's "Précession des Equinoxes" (1749), Giulio Mozzi's theory of finite motions (1763) and Lagrange's memoir on the motion of a rigid body not subject to any net forces or torques (1773).

LARRY D'ANTONIO, Ramapo College

Mating Griffins with Horses: Euler and Kant

Abstract: In a famous remark Kant noted that it would be easier to mate griffins with horses than to unite metaphysics and mathematics. How aware was Kant of Euler's work and how much influence did it have on Kant's development as a philosopher? In this talk we argue that Euler and his Tischgesellschaft (his Berlin circle) did impact the philosophy of Kant. We consider Kant's metaphysics of space and time, his theory of matter, and his adoption of the theory of epigenesis in light of the work of Euler and his disciples Johann Bernhard Merian and Caspar Friedrich Wolff.

STEPHEN DONAHUE, Rowan University

E685: An analytic exercise in which a most general summation of series is found

Abstract: In this paper, which was translated from Latin to English, Euler begins by considering the problem of calculating the arclength of the equilateral hyperbola. After making some observations, Euler arrives at a summation of series. He then generalizes the summation and arrives at what we refer to as the 2F1 hypergeometric series. He then shows how a transform of variables on the series can produce a special case of the binomial series. Lastly, he develops a final generalized series and discusses some of its properties.

KENNETH GITTELSON, Benjamin N. Cardozo High School and Queensborough Community College

Euler and the design of duplicate bridge tournaments

Abstract: This talk will focus on Howell movements with a small number of tables, orthogonal Latin squares, and balanced incomplete block designs.

STACY LANGTON, University of San Diego

A Buckling Paradox

Abstract: Late in his career, Euler wrote a sequence of three papers, E508, 509, 510 (published consecutively in the St. Petersburg Acta for 1778), that deal with the buckling of a heavy column under its own weight. In E508, Euler reached the conclusion that such a column can never buckle under its own weight, no matter how tall it is. In E509 he sees that this conclusion is clearly absurd, but is not yet sure how to resolve the paradox. Finally, in E510, he finds the source of his mistake and arrives at the correct analysis.

These papers contain a wealth of ideas and results, of which I will try to present a sample in my talk.

Thomas Osler, Rowan University

Another Look at Euler's Oblique-angled Diameters

Abstract: In the paper E83, (On some properties shared between conic sections and infinitely many other curves), Euler was examining properties of the conic sections that could be shared by more general curves. Most of the paper is concerned with "oblique-angle diameters", a concept that seems to have been familiar to his readers in the eighteenth century, but has been ignored today.

Definition: Given a curve, and the line (diameter) ET , with an associated number m. Then the line ET is called an *oblique-angled diameter* for this curve, if every chord with slope m is bisected by the diameter ET. If the diameter and curve intersect at T where a well defined tangent line exists, then the slope of this tangent line is the number m.

It turns out that every line parallel to the axis of a parabola is an oblique angled diameter, and every line through the center of an ellipse or a hyperbola is also an oblique angled diameter. This was known to the ancients. Euler finds other curves with more than one oblique angled diameter. We consider the same problem and give a new way of deriving results.

Bruce Petrie, Institute for the History and Philosophy of Science and Technology, University of Toronto

Leonhard Euler's Use and Understanding of Mathematical Transcendence

Abstract: Paul Erdös and Underwood Dudley (1983) suggested that Leonhard Euler was the first to define transcendental numbers as numbers which are not roots of algebraic equations. In contrast to Feldman and Shidlovskii's (1967) claim that major achievements in the theory of transcendental numbers were linked to the emergence of new mathematical methods, Erds and Dudley suggested that Joseph Liouville's (1844) proof of the existence of transcendental numbers was well within Euler's reach in the eighteenth century. The paper analyzes these claims in relation to Euler's original use and apparent understanding of mathematical transcendence.

Kim Plofker (presenting) and Molly Maguire, Union College

Euler's reinvention (and generalization) of spherical trigonometry

Abstract: In the intervals of his prolific groundbreaking work in new and exciting fields of mathematics, Euler made several interesting contributions to old and boring fields of mathematics. In particular, he revisited the topic of spherical trigonometry, whose results had been firmly established and fully understood about two thousand years before his time, and re-derived those results using an approach based on infinitesimal analysis or the method of maxima and minima. This reinvention served as the basis for his subsequent work on generalizing spherical trigonometry to non-spherical surfaces. This paper, which grew out of the second author's senior thesis project and subsequent independent study to translate and analyze E214 and E215 for the Euler Archive, examines why Euler considered it important to re-do spherical trigonometry from scratch, and what these treatises tell us about his perspective on infinitesimal methods.

Michael Saclolo, St. Edward's University
Euler on cases when $x^4 + mx^2y^2 + x^4$ can be reduced to a square.

Abstract: Euler wrote two papers, written about five years apart, on the problem of whether the form $x^4 + mx^2y^2 + x^4$ can be reduced to a square (E696 and E755). In both, Euler starts by mentioning the more trivial cases then proceeds to find admissible values for the variables involved. In the former, Euler provides a list of values for m less than 100 along with the proper ratio between x and y. In the latter work, Euler augments his list and closes with what he calls an "elegant method" of solving the problem.

Erik Tou, Carthage College
Opera Omnia Anglice Reddit

Abstract: As the Euler Archive begins its eighth year, the most exciting new scholarly output comes in the form of new English translations of Euler's works. These range from the very well-known (e.g., Ian Bruce's translation of the *Institutionum Calculi Integralis*) to some uncommon gems (e.g., Saclolo and Wake's translation of *Reflexions sur l'espace et le tems*), and everything in between. In the last decade, the number of translated works has jumped from 10-20 in 2000 to about 140 in 2010. During the last three years, about 60 new translations have become available. At a rate of 30 per year, it is reasonable to project that all of Euler's works could reasonably be translated by 2033, the 250th anniversary year of Euler's death. In this talk, we will look at the role of the Euler Archive and of the myriad translators in this process, and issue a challenge for the future: *ad fontes*!