Math Stories and Histories

How Historical Narratives Shape Mathematical Ideas

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A PROBLEM TO PLAY WITH

What is the largest number of regions into which you can divide a circle using 5 points? What about 6?
THE UNIVERSALITY OF MATHEMATICS

- Everyone does math.
  Mathematical ideas have been pursued in all cultures, times, and places.

- Everyone discovers new ideas.
  The same mathematical idea often appears independently in multiple cultures, times, and places.

- Everyone recognizes a good idea.
  Mathematical ideas are transmitted across and between cultures, times, and places.
The Fibonacci sequence

1, 1, 2, 3, 5, __, __, __, ...

QUIZ TIME!
QUIZ TIME!

1, 1, 2, 3, 5, 8, 13, 21, ...
QUIZ TIME!

1, 1, 2, 3, 5, 8, 13, 21, …

The Fibonacci sequence
QUIZ TIME!

Pascal's triangle

...
QUIZ TIME!

Pascal's triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
...

[Diagram of Pascal's triangle]
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
...

Pascal’s triangle
Pythagorean theorem

3

4

c = ?
Pythagorean theorem

\[ c = \sqrt{4^2 + 3^2} \]
\[ = \sqrt{16 + 9} = \sqrt{25} = 5 \]

QUIZ TIME!

3

4

\( c \)
Pythagorean theorem

\[ c^2 = a^2 + b^2 \]
HISTORICAL TRAJECTORIES
HOW WE TELL THE STORY OF MATHEMATICS

The “classical” trajectory:

Greek and Greco-Roman Culture
- c. 600 BCE
- c. 400 CE

European Renaissance
- c. 1400
- c. 1550

Dark Ages

Scientific Revolution

Modern Europe
- c. 1750
- 1914
PARADIGMS AND CONSEQUENCES

The classical trajectory:
- Flattens the story of math into a single narrative
- Neglects important developments outside Europe during the “Dark Ages”
- Privileges printed/published work
- Presents the history of math in a geographical bubble
- Contributes to a “heroic genius” theory of discovery
Table of Contents from W. W. Rouse Ball’s *A Short Account of the History of Mathematics* (1908):

1. Mathematics Under Greek Influence
   - The Ionian and Pythagorean Schools
   - The Schools of Athens and Cyzicus
   - The First Alexandrian School
   - The Second Alexandrian School
   - The Byzantine School
   - Systems of Numeration and Primitive Arithmetic
A FLATTENED NARRATIVE

Table of Contents from W. W. Rouse Ball’s *A Short Account of the History of Mathematics* (1908):

2. Mathematics of the Middle Ages and the Renaissance
   - The Rise of Learning in Western Europe
   - The Mathematics of the Arabs
   - Introduction of Arabian Works Into Europe
   - The Development of Arithmetic
   - The Mathematics of the Renaissance
   - The Close of the Renaissance
A FLATTENED NARRATIVE

Table of Contents from W. W. Rouse Ball’s *A Short Account of the History of Mathematics* (1908):

3. Modern Mathematics
   - History of Mathematics from Descartes to Huygens
   - The Life and Works of Newton
   - Leibniz and the Mathematicians of the First Half of the Eighteenth Century
   - Lagrange, Laplace, and Their Contemporaries
   - Mathematics of the Nineteenth Century
A FLATTENED NARRATIVE

Some Problematic Chapters and Sections:

- **Systems of Numeration and Primitive Arithmetic**
  - 2 pages on numeration outside Greco-Roman world

- **The Mathematics of the Arabs**
  - 8 pages on India, 7 pages on Arab world

- **Introduction of Arabian Works Into Europe**
  - 2 pages covering Arab writers

- **Note on ignorance of mathematics shewn by the Chinese**

- **Adoption of Arabic system of notation among civilized races**
A FLATTENED NARRATIVE

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“The first great mathematician of the 13th century, and indeed the greatest and most productive mathematician of all the Middle Ages, was Leonardo Fibonacci.”
Smith on Fibonacci’s life and work:

- As son of a customs official, he “... received his early education from a Moorish schoolmaster.”
- “... meeting with scholars and becoming acquainted with the various arithmetic systems in use among the merchants of different lands.”
- “All the systems of computation he counted as poor, however, compared with the one that used our modern numerals.”
- “... wrote a work in 1202, Liber Abaci, in which he gave a satisfactory treatment of arithmetic and elementary algebra.”
Smith’s narrative of Fibonacci:

- Neglects important developments outside Europe
- Privileges printed/published work
- Contributes to a “heroic genius” theory of discovery

Smith’s summary of Fibonacci: “The son of a provincial official became the greatest medieval mathematician.”
Originated by 300s CE
Had symbols for 1–9, also 10, 20, ..., 90.
Symbol for zero emerged in 600s CE.

Simple grouping system: no place values.
Source material: stone inscriptions, birch bark.
TOWARD A BROADER VIEW
CALCULATION IN CHINA

- Counting board & rods
- Dating from 300s BCE

Short sticks arranged horizontally or vertically to represent digits 1-9.

- Square slots provided positions for decimal place values.
- Made for easy arithmetic calculations.
A MODIFIED TRAJECTORY
EXPANDING THE NARRATIVE

Egypt and Mesopotamia

China, India, Arab world

Modern Europe

Greek and Greco-Roman Culture

European Renaissance

Dark Ages

Scientific Revolution
The modified trajectory:

- Still tells the story of math as a single narrative
- Incorporates developments outside Europe from before and after the Greco-Roman period
- Emphasizes shared ideas and efforts
- Often views non-European contributions in terms of how they supported European mathematics
600s CE: Brahmi numerals and Chinese counting board combined form and function to create Hindu numerals.

By the 700s CE: system was adopted & modified by Arab scholars.

Hindu numerals

East Arabic numerals

West Arabic numerals
750s CE: Abbasid caliphate established, with capital at Baghdad

800s CE: Caliph al-Rashid founded House of Wisdom (Bayt al-Hikmah)

Collected knowledge from East & West, translated many works into Arabic.

Caliph al-Mansur invited Indian scholars to study in Baghdad.

Caliph al-Ma’mun sent an embassy to Constantinople specifically to obtain a copy of Euclid’s Elements.
c. 800 CE: al-Khwarizmi writes *The Compendious Book on Calculation by Completion and Balancing*
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Completion → al-ğabr
→ algebra

al-Khwarizmi → algorismi
→ algorithm
ALGORITHMS AND ALGEBRA
THE ISLAMIC GOLDEN AGE

- c. 800 CE: al-Khwarizmi writes *The Compendious Book on Calculation by Completion and Balancing*
  - Completion → al-ğabr
  - al-Khwarizmi → algorismi
- Hindu numerals allowed for systematic and abstract calculations
- Euclidean geometry and logic allowed for abstract theorems and proofs
LIGHTNING ROUND!

Fibonacci sequence?

1, 1, 2, 3, 5, 8, 13, 21, …
Known to Acarya Virahanka (India, c. 700 CE).

Sanskrit poetry used short (|) and long (S) syllables to construct metrical patterns.

“The variations of two earlier meters being mixed, the number is obtained.”
Pascal’s triangle?
Yang’s triangle?

Described by Yang Hui (China, 1261 CE)

Used the triangle as a tool to compute binomial expansions.

“Add the numbers in the two places above in order to find the number in the pace below.”
Pythagorean theorem?

\[ a^2 + b^2 = c^2 \]
LIGHTNING ROUND!

**Euclid** (Egypt, c. 300 BCE) gave a deductive proof of it in the *Elements*.

**Zhao Shuang** (China, 200s CE) gave an argument for it in commentary on the *Arithmetical Classic of the Gnomon*.

Examples of “Pythagorean” triangles from Babylonian tablets (c. 1800 BCE), Sanskrit *Sulbasutras* (c. 800 BCE), and elsewhere…
EXPAND THE NARRATIVE?
EXPAND THE NARRATIVE?
WHERE DO WE GO FROM HERE?
A GLOBAL HISTORY OF MATHEMATICS

- There needs to be a narrative, but not everything must be a narrative (and there can be more than one!).
- Think of history not as a straight line of discovery or a set of biographies, but instead as a network of ideas.
- Culture and geography will play important roles, although boundaries are porous.
- Students exposed to a global story of mathematics develop a broader sense of who can do math and for what purpose.
- Math is a team effort: cooperation over competition
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<th>points</th>
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<td>31</td>
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For $n$ points: \( \frac{n^4 - 6n^3 + 23n^2 - 18n + 24}{24} \) regions