UNIVERSITY OF CHICAGO
Graduate School of Business

Business 333
Corporation Finance

Larry Y. Dann
Spring, 1984

STATISTICS REFRESHER FOR MEANS, VARIANCES AND COVARIANCES

$\hat{X}$, $\hat{Y}$, $\hat{Z}$ $\equiv$ random variables

a, b, c, d $\equiv$ constants

$\text{Var}(\hat{X}) = \sigma_{xx} = \sigma_x^2$ variance of the random variable $\hat{X}$

$\text{Cov}(\hat{X}, \hat{Y}) = \sigma_{xy} \equiv$ covariance of $\hat{X}$ and $\hat{Y}$

I. Expected value or mean

$E(\hat{X}) \equiv$ the expected value of the random variable $\hat{X}$

$$E(\hat{X}) = p_1X_1 + p_2X_2 + \ldots + p_nX_n$$

here there are n possible values of X, each with a probability p of occurrence

$$= \sum_{j=1}^{n} p_jX_j$$

II. Variance

$$\text{Var}(\hat{X}) = \sigma_{xx} = \sigma_x^2 = E\{[\hat{X} - E(\hat{X})]^2\}$$

= the <u>variance</u> of the random variable $\hat{X}$

= the expected squared deviation from the mean

$$= E(\hat{X}^2) - [E(\hat{X})]^2$$

$$= \sum_{i=1}^{n} p_i[\hat{X}_i - E(\hat{X})]^2$$

Note: $\sigma_x = \{\sum_{i=1}^{n} p_i[\hat{X}_i - E(\hat{X})]^2\}^{1/2}$ is the <u>standard deviation</u> of $\hat{X}$

III. Covariance

With two random variables $\hat{X}$ and $\hat{Y}$, we must have joint probabilities

$$p_{ij} \equiv \text{Prob}(\hat{X} = X_i \text{ and } \hat{Y} = Y_j)$$

2

$$\mathrm{Cov}(\hat{X}, \hat{Y}) = \sigma_{xy} = \sigma_{yx} = \mathrm{Cov}(\hat{Y}, \hat{X}) \quad = \rho(\tilde{r}_1, \tilde{r}_2)\sigma_1\sigma^2$$

$$= E[\hat{X} - E(\hat{X})][(\hat{Y} - E(\hat{Y})]$$

$$= E(\hat{X}\hat{Y}) - E(\hat{X})E(\hat{Y})$$

= the <u>covariance</u> of $\hat{X}$ and $\hat{Y}$

= the expected cross-product of deviations about the means

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} p_{ij}[X_i - E(\hat{X})][Y_j - E(\hat{Y})]$$

Note: $\mathrm{Cov}(\hat{X}, \hat{X}) = \sigma_{xx} = \sigma_x^2 = \mathrm{Var}(\hat{X})$

Note: The <u>correlation coefficient</u> of $\hat{X}$ and $\hat{Y}$ is

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{\mathrm{Cov}(\hat{X}, \hat{Y})}{\sigma_x \sigma_y}$$

= the covariance of $\hat{X}$ and $\hat{Y}$ divided by the standard deviations of $\hat{X}$ and $\hat{Y}$

IV. Rules for means, variances and covariances

1. $E(a\hat{X} + b) = aE(\hat{X}) + b$
2. $\mathrm{Var}(a\hat{X}) = a^2\ \mathrm{Var}(\hat{X})$
3. $\mathrm{Var}(\hat{X} + b) = \mathrm{Var}(\hat{X})$
4. $\mathrm{Var}(a\hat{X} + b) = a^2\ \mathrm{Var}(\hat{X})$ (combines Rules 2 and 3)
5. $\mathrm{Cov}(a\hat{X} + b, c\hat{Y} + d) = ac\mathrm{Cov}(\hat{X}, \hat{Y})$
6. $\mathrm{Cov}(b, c\hat{Y} + d) = 0$

V. Sums of random variables

1. $E[a\hat{X} + b\hat{Y} + c\hat{Z}] = aE(\hat{X}) + bE(\hat{Y}) + cE(\hat{Z})$
2. $\mathrm{Var}[a\hat{X} + b\hat{Y} + c\hat{Z}] = a^2\ \mathrm{Var}(\hat{X}) + b^2\ \mathrm{Var}(\hat{Y}) + c^2\ \mathrm{Var}(\hat{Z}) + 2ab\mathrm{Cov}(\hat{X}, \hat{Y}) + 2ac\mathrm{Cov}(\hat{X}, \hat{Z}) + 2bc\mathrm{Cov}(\hat{Y}, \hat{Z})$
3. $\mathrm{Cov}[a\hat{X} + b\hat{Y}, c\hat{Z}] = ac\mathrm{Cov}(\hat{X}, \hat{Z}) + bc\mathrm{Cov}(\hat{Y}, \hat{Z})$