


A. Sampling from the Posterior

We use Gibbs sampling to sample from the joint posterior distribution of the parameters: $\beta$, $\alpha$, $\gamma$, $\lambda$, $b_0$, and $V_0$. The joint posterior does not have a closed form; however, given that the conditional posteriors either have a closed form or are log-concave, implementation of the Gibbs sampler is straightforward. Let $D$ denote the data and $\text{rest}$ denote the remaining parameters. Then at each iteration of the Gibbs sampler, we proceed as follows:

1. Let $\beta_{il} = (\beta_{i0}, \ldots, \beta_{ilp})$, $i = 1, \ldots, N$, $l = 1, \ldots, q$. Then use ARS to sample $[\beta_{il}|\text{rest}, D]$ from

$$
p(\beta_{il}|\text{rest}, D) \propto \exp \left\{ -\frac{1}{2} \left[ \beta_{il}' \left( \sum_{j=1}^{m_i} B_{ij}(t_{ij})^2 \Sigma^{-1} + V_{0l}^{-1} \right) \beta_{il} 
\right.
\right.
\left. - 2\beta_{il}' \left( \Sigma^{-1} A_{il} + V_0^{-1}(b_{0l} + x_i'\alpha) \right) \right]\left. \right\}
\left. \times \exp \left\{ \nu_i(\gamma'\psi\beta(s_i) + z_i'\zeta) - e^{z_i'\zeta} \sum_{j=1}^{J} H_{ij}(\beta, \gamma, \lambda) \right\},
\right.
$$
where

$$A_{il} = \sum_{j=1}^{m_i} \begin{pmatrix} Y_{ij1} - \sum_{k \neq l}^{q} \beta_{ik1} B_k(t_{ij}) \\ \vdots \\ Y_{ijp} - \sum_{k \neq l}^{q} \beta_{ikp} B_k(t_{ij}) \end{pmatrix}.$$  

2. Sample

$$[V_{0l}^{-1}|\text{rest}, D] \sim \text{Wishart}\left(\left(S_{\text{vol}}^{-1} + \sum_{i=1}^{N}(\beta_{il} - b_{0l} - x_i'\alpha)(\beta_{il} - b_{0l} - x_i'\alpha)'\right)^{-1}, N + \nu_{\text{vol}}\right).$$

3. Sample

$$[\Sigma^{-1}|\text{rest}, D] \sim \text{Wishart}\left(\left(S^{-1} + \sum_{i=1}^{N} \sum_{j=1}^{m_i} (Y_{ij} - \psi_{\beta}(t_{ij}))(Y_{ij} - \psi_{\beta}(t_{ij}))'\right)^{-1}, \sum m_i + \nu_e\right).$$

4. Let $b_{0l} = (b_{0l0}, \ldots, b_{0lp})'$, $i = 1, \ldots, N$, $l = 1, \ldots, q$. Then sample

$$[b_{0l}|\text{rest}, D] \sim N_2(\mu_{b_{0l}}, \Sigma_{b_{0l}}),$$

where

$$\mu_{b_{0l}} = \Sigma_{b_{0l}}(V_{0l}^{-1} \sum_{i=1}^{N}(\beta_{il} - x_i'\alpha) + A_{1l}^{-1}A_0)$$

and

$$\Sigma_{b_{0l}} = (NV_{0l}^{-1} + A_{1l}^{-1})^{-1}.$$  

5. Use ARS to sample $[\gamma|\text{rest}, D]$ from

$$p(\gamma|\text{rest}, D) \propto \exp\left\{\sum_{i=1}^{N}(\nu_i(\gamma'\psi_{\beta}(s_i) + z_i'\zeta) - e^{z_i'\zeta} \sum_{j=1}^{J} H_{ij}(\beta, \gamma, \lambda))\right\} \times \exp\left\{-\frac{1}{2}(\gamma - g_0)'g_1^{-1}(\gamma - g_0)\right\}.$$  

23
6. Sample $[\lambda_j | \text{rest}, D] = \text{gamma}(d_{0j} + n_j, \sum_{i=1}^{N} e^{\beta l} H_{ij}(\beta, \gamma, 1) + d_{1j})$, where $n_j$ is the number of events in the $j$th interval and $H_{ij}(\beta, \gamma, 1)$ is $H_{ij}(\beta, \gamma, \lambda)$ evaluated with $\lambda_j = 1$.

7. Sample $[\alpha | \text{rest}, D] \sim N_p(\mu_\alpha, \Sigma_\alpha)$, where

$$\mu_\alpha = \sum_{i=1}^{N} x_i \sum_{l=1}^{q} V_{0l}^{-1}(\beta_{il} - b_{0l}) + C_1^{-1} C_0$$

and

$$\Sigma_\alpha = \left(q \sum_{i=1}^{N} x_i V_{0l}^{-1} x_i' + C_1^{-1}\right)^{-1}.$$

B. Estimating the predicted survival curve

Given the $G$ MCMC samples, $b_{0l}^{(g)}, \alpha^{(g)}, \lambda^{(g)}, \gamma^{(g)}$ and $V_{0l}^{(g)}, g = 1, \ldots, G$, we can estimate the predicted survival curve at a set of time points, $u = u_1, \ldots, J$, as follows.

Repeat the following steps for $g = 1, \ldots, G$.

1. Sample $\beta_{1,1}^{*} \sim N_2(b_{0l}^{(g)} + \alpha^{(g)}, V_{0l}^{(g)})$ and $\beta_{1,2}^{*} \sim N_2(b_{0l}^{(g)}, V_{0l}^{(g)})$.

2. Calculate $S_k^{(g)}(u_j) = \exp\left\{- \int_0^{u_j} \lambda^{(g)}(t) e^{\gamma^{(g)}(t) x(t) + \zeta^{(2-k)}} dt\right\}$, $k = 1, 2$.

The predicted survival curve at time $u_j$ is then, $S_k(u_j) = \text{median}(S_k^{(g)}(u_j), g = 1, \ldots, G)$, $k = 1, 2$, where $k = 1$ indicates the treatment group and $k = 2$ is the non-treatment group.