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### A. Sampling from the Posterior

We use Gibbs sampling to sample from the joint posterior distribution of the parameters:  $\beta$ ,  $\alpha$ ,  $\gamma$ ,  $\lambda$ ,  $b_o$ , and  $V_0$ . The joint posterior does not have a closed form; however, given that the conditional posteriors either have a closed form or are log-concave, implementation of the Gibbs sampler is straight-forward. Let  $D$  denote the data and  $rest$  denote the remaining parameters. Then at each iteration of the Gibbs sampler, we proceed as follows:

1. Let  $\beta_{il} = (\beta_{il0}, \dots, \beta_{ilp})'$ ,  $i = 1, \dots, N$ ,  $l = 1, \dots, q$ . Then use ARS to sample  $[\beta_{il}|rest, D]$  from

$$\begin{aligned}
 p(\beta_{il}|rest, D) \propto & \exp \left\{ -\frac{1}{2} \left[ \beta'_{il} \left( \sum_{j=1}^{m_i} B_l(t_{ij})^2 \Sigma^{-1} + V_{0l}^{-1} \right) \beta_{il} \right. \right. \\
 & \left. \left. - 2\beta'_{il} (\Sigma^{-1} A_{il} + V_0^{-1} (b_{0l} + x'_i \alpha)) \right] \right\} \\
 & \times \exp \left\{ \nu_i (\gamma' \psi_\beta(s_i) + z'_i \zeta) - e^{z'_i \zeta} \sum_{j=1}^J H_{ij}(\beta, \gamma, \lambda) \right\},
 \end{aligned}$$

where

$$A_{il} = \sum_{j=1}^{m_i} \begin{pmatrix} Y_{ij1} - \sum_{\substack{k=1 \\ k \neq l}}^q \beta_{ik1} B_k(t_{ij}) \\ \vdots \\ Y_{ijp} - \sum_{\substack{k=1 \\ k \neq l}}^q \beta_{ikp} B_k(t_{ij}) \end{pmatrix}.$$

## 2. Sample

$$[V_{0l}^{-1} | rest, D] \sim Wishart \left( \left( S_{v_{0l}}^{-1} + \sum_{i=1}^N (\beta_{il} - b_{0l} - x'_i \alpha)(\beta_{il} - b_{0l} - x'_i \alpha)' \right)^{-1}, N + \nu_{v_{0l}} \right).$$

## 3. Sample

$$[\Sigma^{-1} | rest, D] \sim Wishart \left( \left( S^{-1} + \sum_{i=1}^N \sum_{j=1}^{m_i} (Y_{ij} - \psi_{\beta}(t_{ij}))(Y_{ij} - \psi_{\beta}(t_{ij}))' \right)^{-1}, \sum m_i + \nu_{\epsilon} \right).$$

4. Let  $b_{0l} = (b_{0l0}, \dots, b_{0lp})'$ ,  $i = 1, \dots, N$ ,  $l = 1, \dots, q$ . Then sample

$$\begin{aligned} [b_{0l} | rest, D] &\sim N_2(\mu_{b_{0l}}, \Sigma_{b_{0l}}), \text{ where} \\ \mu_{b_{0l}} &= \Sigma_{b_{0l}} (V_{0l}^{-1} \sum_{i=1}^N (\beta_{il} - x'_i \alpha) + A_1^{-1} A_0) \text{ and} \\ \Sigma_{b_{0l}} &= (NV_{0l}^{-1} + A_1^{-1})^{-1}. \end{aligned}$$

5. Use ARS to sample  $[\gamma | rest, D]$  from

$$\begin{aligned} p(\gamma | rest, D) &\propto \exp \left\{ \sum_{i=1}^N \left( \nu_i (\gamma' \psi_{\beta}(s_i) + z_i' \zeta) - e^{z_i' \zeta} \sum_{j=1}^J H_{ij}(\beta, \gamma, \lambda) \right) \right\} \times \\ &\exp \left\{ -\frac{1}{2} (\gamma - g_0)' g_1^{-1} (\gamma - g_0) \right\}. \end{aligned}$$

6. Sample  $[\lambda_j | rest, D] = \text{gamma}(d_{0j} + n_j, \sum_{i=1}^N e^{z'_i \zeta} H_{ij}(\beta, \gamma, 1) + d_{1j})$ , where  $n_j$  is the number of events in the  $j$ th interval and  $H_{ij}(\beta, \gamma, 1)$  is  $H_{ij}(\beta, \gamma, \lambda)$  evaluated with  $\lambda_j = 1$ .

7. Sample

$$[\alpha | rest, D] \sim N_p(\mu_\alpha, \Sigma_\alpha), \text{ where}$$

$$\mu_\alpha = \Sigma_\alpha \left( \sum_{i=1}^N x_i \sum_{l=1}^q V_{0l}^{-1}(\beta_{il} - b_{0l}) + C_1^{-1} C_0 \right) \text{ and}$$

$$\Sigma_\alpha = \left( q \sum_{i=1}^N x_i V_{0l}^{-1} x_i' + C_1^{-1} \right)^{-1}.$$

## B. Estimating the predicted survival curve

Given the  $G$  MCMC samples,  $b_0^{(g)}, \alpha^{(g)}, \lambda^{(g)}, \gamma^{(g)}$  and  $V_0^{(g)}, g = 1, \dots, G$ , we can estimate the predicted survival curve at a set of time points,  $u = u_1, \dots, J$ , as follows.

Repeat the following steps for  $g = 1, \dots, G$ .

1. Sample  $\beta_{l,1}^* N_2(b_{0l}^{(g)} + \alpha^{(g)}, V_{0l}^{(g)})$  and  $\beta_{l,2}^* N_2(b_{0l}^{(g)}, V_{0l}^{(g)})$ .
2. Calculate  $S_k^{(g)}(u_j) = \exp\{-\int_0^{u_j} \lambda^{(g)}(t) e^{\gamma'(g)' \psi_\beta(t) + \zeta^*(2-k)} dt\}$ ,  $k = 1, 2$ .

The predicted survival curve at time  $u_j$  is then,  $S_k(u_j) = \text{median}(S_k^{(g)}(u_j), g = 1, \dots, G)$ ,  $k = 1, 2$ , where  $k = 1$  indicates the treatment group and  $k = 2$  is the non-treatment group.