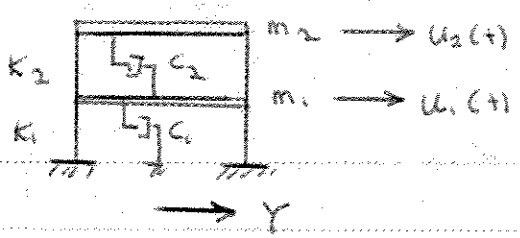
Support Motion



EOM -

mass 1 ① $m_1 \ddot{u}_1 = -c_1(\dot{u}_1 - \dot{Y}) - k_1(u_1 - Y) + c_2(\dot{u}_2 - \dot{u}_1) + k_2(u_2 - u_1)$

mass 2 ② $m_2 \ddot{u}_2 = -c_2(\dot{u}_2 - \dot{u}_1) - k_2(u_2 - u_1)$

Let $z_1 = u_1 - Y$, $\dot{z}_1 = \dot{u}_1 - \dot{Y}$, $\ddot{z}_1 = \ddot{u}_1 - \ddot{Y}$

$z_2 = u_2 - Y$, $\dot{z}_2 = \dot{u}_2 - \dot{Y}$, $\ddot{z}_2 = \ddot{u}_2 - \ddot{Y}$

①′ $m_1 \ddot{z}_1 + c_1 \dot{z}_1 + k_1 z_1 - c_2(\dot{z}_2 - \dot{z}_1) - k_2(z_2 - z_1) = -m_1 \ddot{Y}$

②′ $m_2 \ddot{z}_2 + c_2(\dot{z}_2 - \dot{z}_1) + k_2(z_2 - z_1) = -m_2 \ddot{Y}$

In matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix}$$

$$= -\ddot{Y} \begin{Bmatrix} m_1 \\ m_2 \end{Bmatrix} = -\ddot{Y} [M] \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

In modal coordinates

$$m^*_{jj} \ddot{q}_j + 2 c^*_{jj} \dot{q}_j + k^*_{jj} q_j = -\ddot{Y} \{\phi_j\}^T [M] \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

or equivalently

$$\ddot{q}_j + 2 \omega_j \zeta_j \dot{q}_j + \omega_j^2 q_j = \underbrace{\frac{\{\phi_j\}^T [M] \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}}{m^*_{jj}}}_{\text{modal participation}} (-\ddot{Y})$$