Solution Strategy for Slender Beams

1) Obtain EOM $$m\left(\frac{\partial^2 v}{\partial t^2}\right) + \frac{\partial^2}{\partial x^2}\left(EI\,\frac{\partial^2 v}{\partial x^2}\right) = f(x,t)$$

or $$m\ddot{v} + (EI\,v'')'' = f$$

2) Determine mode shapes and frequencies by considering free vibration

(Uniform beam, EI constant) $$\psi_m'''' - \frac{m\omega_m^2}{EI}\psi_m = 0$$

3) Determine uncoupled equations

$$m^*_m \ddot{q}_m + k^*_m q_m = f^*_m$$

where $$m^*_m = \int_0^\ell m\,\psi_m^2\,dx$$

$$k^*_m = \int_0^\ell EI(\psi_m'')^2\,dx$$

$$f^*_m = \int_0^\ell f\,\psi_m\,dx$$

4) Determine initial conditions in terms of q.

$$v(x,0) = \sum_{m=1}^{\infty} \psi_m(x)\,q_m(0)$$

$$\dot{v}(x,0) \quad \sum_{m=1}^{\infty} \psi_m(x)\,\dot{q}_m(0)$$

5) Solve as many SDOF problems as desired $q_1 \cdots q_N$

6) Convert back to original coordinate system $$v(x,t) = \sum_{m=1}^{N} \psi_m(x)\,q_m(t)$$