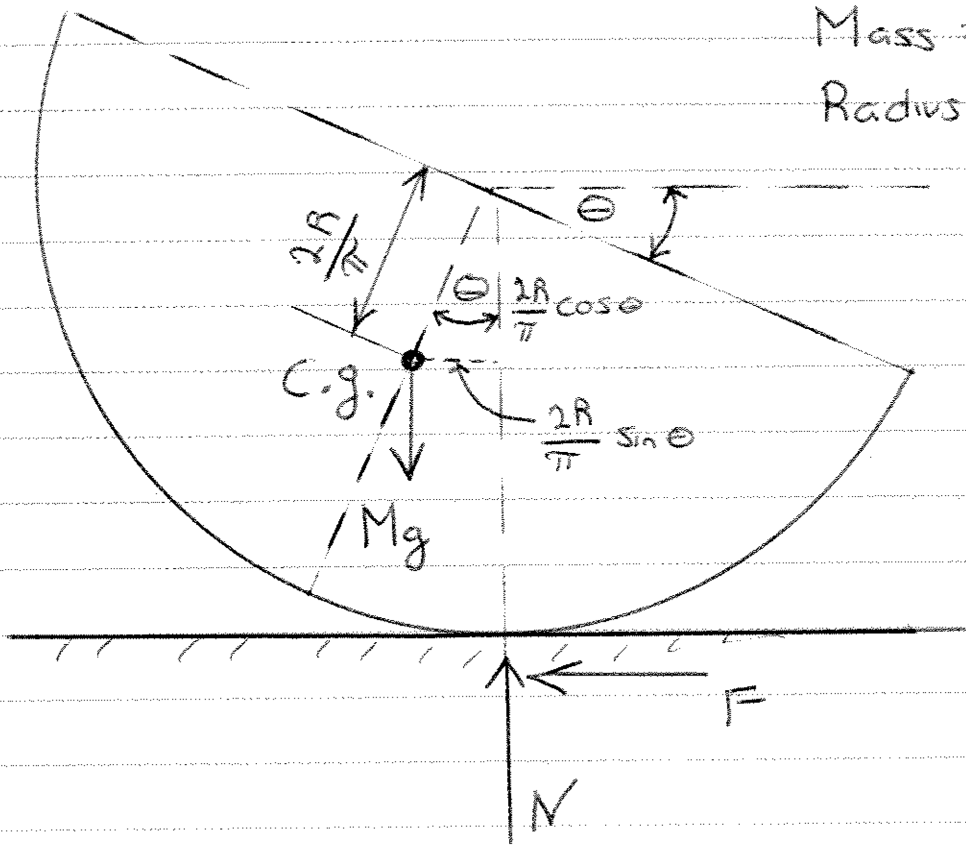
EOM Example

Rocking of Semi circular Thin Plate



Using Linear + Angular Momentum Relationships

1) $F_{ext_x} = -F = Ma_x = M\left(\ddot{\theta}\left(R - \frac{2R}{\pi}\cos\theta\right)\right)$

2) $F_{ext_y} = N - Mg = Ma_y = M\left(\ddot{\theta}\,\frac{2R}{\pi}\sin\theta\right)$

3) $M_{cm} = F\left(R - \frac{2R}{\pi}\cos\theta\right) - N\left(\frac{2R\sin\theta}{\pi}\right) = I_{zz}\ddot{\theta}$

where

$$I_{zz_{cm}} = I_{\substack{\text{center}\\ \text{of circle}}} - M\bar{y}^2$$

$$= MR^2 - M\left(\frac{2R}{\pi}\right)^2 = MR^2\left(1 - \frac{4}{\pi^2}\right)$$

Substitute ① & ② into ③

$$\underbrace{-M\ddot{\theta}\left(R-\frac{2R}{\pi}\cos\theta\right)}_{F}\left(R-\frac{2R}{\pi}\cos\theta\right)$$

$$-\underbrace{\left(M\ddot{\theta}\frac{2R}{\pi}\sin\theta+Mg\right)}_{N}\left(\frac{2R\sin\theta}{\pi}\right)=MR^2\left(1-\frac{4}{\pi^2}\right)\ddot{\theta}$$

$$\ddot{\theta}R^2\left[-1+\frac{4}{\pi}\cos\theta-\frac{4}{\pi^2}\cos^2\theta-\frac{4}{\pi^2}\sin^2\theta-1+\frac{4}{\pi^2}\right]=\frac{-2gR\sin\theta}{\pi}$$

$$\ddot{\theta}R^2\left[-2+\frac{4}{\pi}\cos\theta\right]=\frac{-2gR\sin\theta}{\pi}$$

$$\sin\theta+\left[\pi-2\cos\theta\right]\frac{R}{g}\ddot{\theta}=0$$

For small θ:

$$\boxed{\theta+\left[\pi-2\right]\frac{R}{g}\ddot{\theta}=0}$$

Work & Kinetic Energy

Consider system at 1 $(\theta = 0°)$
and 2 $\theta = \theta(t)$

$$W_{1\to 2} = -Mg\left[\frac{2R}{\pi} - \frac{2R}{\pi}\cos\theta\right] = -\frac{2MgR}{\pi}(1-\cos\theta)$$

@ 1, $T_1 = KE_1$

@ 2 $T_2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{zz}\dot{\theta}^2$

$$= \frac{1}{2}M\dot{\theta}^2\left[\left(\frac{2R}{\pi}\sin\theta\right)^2 + \left(R - \frac{2R}{\pi}\cos\theta\right)^2\right] + \frac{1}{2}MR^2\left(1-\frac{4}{\pi^2}\right)\dot{\theta}^2$$

$$= \frac{1}{2}M\dot{\theta}^2\left[\frac{4R^2}{\pi^2} + R^2 - \frac{4R^2}{\pi}\cos\theta\right] + \frac{1}{2}MR^2\left(1-\frac{4}{\pi^2}\right)\dot{\theta}$$

$$W_{1\to 2} = T_2 - T_1$$

$$-\frac{2MgR}{\pi}(1-\cos\theta) = \overbrace{\frac{1}{2}M\left[\left(R-\frac{2R}{\pi}\right)\dot{\theta}\right]^2}^{\text{small } \theta} + \frac{1}{2}MR^2\left(1-\frac{4}{\pi^2}\right)\dot{\theta}^2 - KE_1$$

Take derivatives respect to time.

$$-\frac{2MgR}{\pi}(\sin\theta)\cancel{\dot{\theta}} = M\left(R-\frac{2R}{\pi}\right)^2\cancel{\dot{\theta}}\ddot{\theta} + MR^2\left(1-\frac{4}{\pi^2}\right)\cancel{\dot{\theta}}\ddot{\theta} - 0$$

$$0 = \left[\frac{2MgR}{\pi}\right]\sin\theta + M\left[2R^2 - \frac{4R^2}{\pi}\right]\ddot{\theta}$$

Let θ be small

$$\boxed{0 = \theta + \frac{R}{g}[\pi - 2]\ddot{\theta}}$$

Same as before.