# EOM for Slender Beams

$$\Sigma F_y = m\ddot{y} \rightarrow m\ddot{y} = f + V$$
$$\Sigma M = I_p\ddot{\theta} = 0 \rightarrow V = M'$$
$$\rightarrow m\ddot{y} - M'' = f \rightarrow \boxed{m\ddot{y} + (EIy'')'' = f}$$

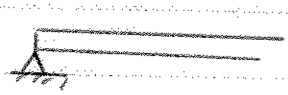
Linear Material Properties
Plane sections remain plane

$$M = -EIy''$$

## Common Boundary Conditions

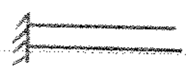
Prime refers to spacial derivative
Dot refers to time derivative.

1) Simply supported



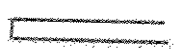
$y(0,t) = 0$
$y''(0,t) = 0$

2 Fixed End



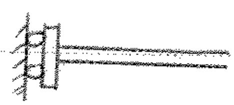
$y(0,t) = 0$
$y'(0,t) = 0$

3 Free End



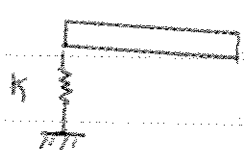
$y''(0,t) = 0$
$y'''(0,t) = 0$

4

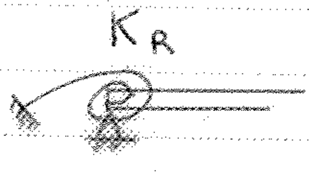


$y'(0,t) = 0$
$y'''(0,t) = 0$

5 Elastic support




$y''(0,t) = 0$
$EIy'''(0,t) = Ky(0,t)$




$y(0,t) = 0$
$EIy''(0,t) = K_R\, y'(0,t)$

## Free - Vibration

We will attempt to find a solution similar to that obtained for a discrete solution in that the spatial and temporal parts of the solution are separable.

Discrete System $\{u\} = \{\phi\} q(t)$

Continuous System $v(x,t) = \psi(x)\, q(t)$

EOM $m\ddot{v} + EI v'''' = 0$ (uniform beam)

$\ddot{v} = \psi \ddot{q}$ $\quad v'''' = \psi'''' q$

$$m\psi\ddot{q} + EI\psi'''' q = 0$$

$$\boxed{\frac{\psi''''}{\psi}\left(\frac{EI}{m}\right) = -\frac{\ddot{q}}{q}}$$

To be valid all times and locations

$$-\frac{\ddot{q}}{q} = \text{constant} = \omega^2 \longrightarrow \boxed{\ddot{q} + \omega^2 q = 0}$$

(Harmonic Motion)

$$\frac{\psi''''}{\psi}(EI/m) = \omega^2 \longrightarrow \boxed{\psi'''' - \frac{m\omega^2}{EI}\psi = 0}$$

(Solve to get mode shape)

# Mode - Shape Calculation

D. Eq $\Rightarrow$ $$\boxed{\psi'''' - \frac{m\omega^2}{EI}\psi = 0}$$

Try :

$$\psi(x) = e^{\Delta x} \longrightarrow \left(\Delta^4 - \frac{m\omega^2}{EI}\right)e^{\Delta x} = 0$$

$$\Delta^4 = \frac{m\omega^2}{EI} = \lambda^4 \qquad \Delta = \pm\lambda \text{ or } \pm i\lambda$$

$$\psi(x) = Ae^{\lambda x} + Be^{-\lambda x} + Ce^{i\lambda x} + De^{-i\lambda x}$$

or equivalently

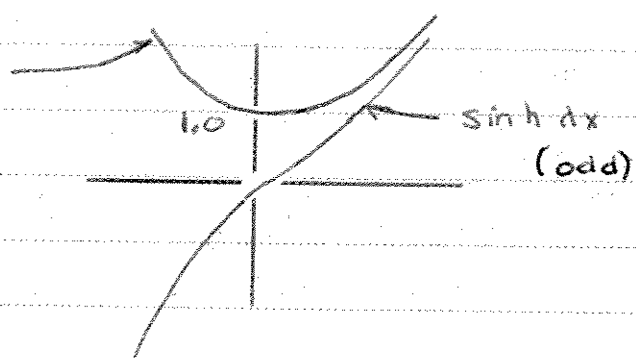
$$\boxed{\psi(x) = C_1\cos\lambda x + C_2\sin\lambda x + C_3\cosh\lambda x + C_4\sinh\lambda x}$$

where :

$$\cosh\lambda x = \tfrac{1}{2}\left(e^{\lambda x} + e^{-\lambda x}\right) \longrightarrow \frac{d}{dx}\cosh\lambda x = \lambda\sinh\lambda x$$

$$\sinh\lambda x = \tfrac{1}{2}\left(e^{\lambda x} - e^{-\lambda x}\right) \longrightarrow \frac{d}{dx}\sinh\lambda x = \lambda\cosh\lambda x$$

cosh λx
(even)




sinh λx
(odd)

Derivatives of $\psi$

$$\psi(x) = C_1\cos\lambda x + C_2\sin\lambda x + C_3\cosh\lambda x + C_4\sinh\lambda x$$

$$\psi'(x) = \lambda\left(-C_1\sin\lambda x + C_2\cos\lambda x + C_3\sinh\lambda x + C_4\cosh\lambda x\right.$$

$$\psi''(x) = \lambda^2\left(-C_1\cos\lambda x - C_2\sin\lambda x + C_3\cosh\lambda x + C_4\sinh\lambda x\right.$$

$$\psi'''(x) = \lambda^3\left(C_1\sin\lambda x - C_2\cos\lambda x + C_3\sin\lambda x + C_4\cosh\lambda x\right.$$