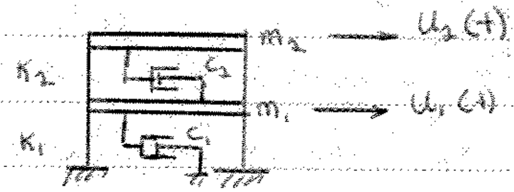
Damping in MDOF Systems

Consider 2 DOF system



Note that the damping is assumed to be proportional to the relative velocity of one floor wrt the other.

EOM -

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\substack{[M] \\ \text{Same as before}}} \underbrace{\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix}}_{\{\ddot{u}\}} + \underbrace{\begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}}_{[C]} \underbrace{\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix}}_{\{\dot{u}\}} + \underbrace{\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}}_{[K]} \underbrace{\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}}_{\{u\}} = \underbrace{\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}}_{\{f\}}$$

As before, let $\{u\} = [\phi]\{q\}$

$$\{\dot{u}\} = [\phi]\{\dot{q}\}$$

$$\{\ddot{u}\} = [\phi]\{\ddot{q}\}$$

and premultiply Eqn. by $[\phi]^T$

$$[M^*]\{\ddot{q}\} + [\phi]^T[C][\phi]\{\dot{q}\} + [K^*]\{q\} = \{f^*\}$$

In general $[\phi]^T[C][\phi]$ is not diagonal.

However, if you choose

$$[C] = \underset{\text{constant}}{\alpha}[M] + \underset{\text{constant}}{\beta}[K]$$

Then $[M^*]\{\ddot{q}\} + [\phi]^T\left(\alpha[M] + \beta[K]\right)[\phi]\{\dot{q}\} + [K^*]\{q\} = \{f^*\}$

$$\boxed{[M^*]\{\ddot{q}\} + \left(\alpha[M^*] + \beta[K^*]\right)\{\dot{q}\} + [K^*]\{q\} = \{f^*\}}$$

Note that the assumption that $[C]$ is a linear combination of $[M]$ and $[K]$ is made only because of convenience. Such damping is referred to as "classical damping" or "proportional damping".

To investigate the significance of the assumption of classical damping, examine the terms of the modal equations.

$$m^*_{jj}\,\ddot{q}_j + \underbrace{\left(\alpha\, m^*_{jj} + \beta\, k^*_{jj}\right)}_{c^*_{jj}}\dot{q}_j + k^*_{jj}\,q_j = f^*_j$$

Introduce definitions that are similar to those used to describe SDOF systems.

$$\zeta_j = \frac{c^*_{jj}}{2\sqrt{m^*_{jj}\,k^*_{jj}}} \qquad \omega_j = \sqrt{\frac{k^*_{jj}}{m^*_{jj}}}$$

$$\boxed{\ddot{q}_j + 2\omega_j\,\zeta_j\,\dot{q}_j + \omega_j^2\,q_j = \frac{f^*_j}{m^*_{jj}}}$$

How does modal damping, ζ_j, vary with α and β?

$$\zeta_j = \frac{\alpha m^*_{jj} + \beta k^*_{jj}}{2\sqrt{m^*_{jj}\,k^*_{jj}}} = \frac{\alpha}{2}\frac{1}{\omega_j} + \frac{\beta}{2}\omega_j$$

<u>Case 1</u> - <u>$\alpha \neq 0,\ \beta = 0$</u> $\longrightarrow$ $\zeta_j = \dfrac{\alpha}{2}\dfrac{1}{\omega_j}$ (Mass-Proportional Damping)

Damping is inversely proportional to modal frequency.
Damping is greater for lower modes than for higher modes.

<u>Case 2</u> - <u>$\alpha = 0,\ \beta \neq 0$</u> $\longrightarrow$ $\zeta_j = \dfrac{\beta}{2}\omega_j$ (Stiffness-Proportional Damping)

Damping is proportional to modal frequency.
Damping is lower for lower modes than for higher modes.

Neither assumption is particularly defensible. Generally, ζ_j are assumed based on measurements or tradition and one works directly with the boxed equation above.