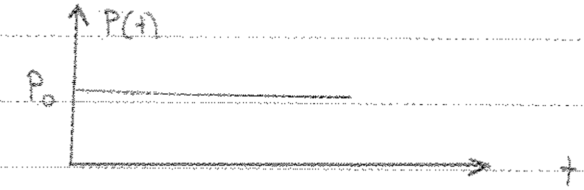
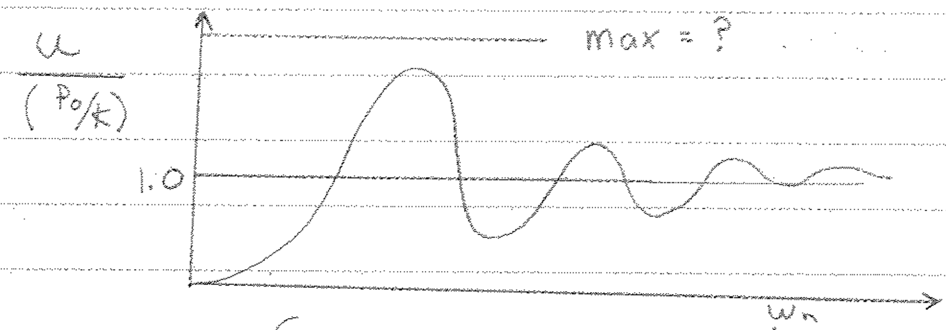
Blast Loading

- a) effect of damping
- b) effect of blast length
- c) example



a) Effect of Damping

$$u(t) = \frac{P_o}{k}\left[1 - e^{-\xi\omega_n t}\left\{\cos\omega_d t + \frac{\xi}{(1-\xi^2)^{1/2}}\sin\omega_d t\right\}\right]$$



$$du/dt = -\frac{P_o}{k}\left\{e^{-\xi\omega_n t}\left[-\omega_d \sin\omega_d t + \overbrace{\frac{\omega_d}{(1-\xi^2)^{1/2}}}^{\omega_n}\xi\cos\omega_d t\right] - \xi\omega_n e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi}{(1-\xi^2)^{1/2}}\sin\omega_d t\right]\right\}$$

(same)

$$= \frac{P_o}{K}e^{-\xi\omega_n t}\left[\omega_d + \xi\omega_n\frac{\xi}{(1-\xi^2)^{1/2}}\right]\sin\omega_d t$$

$\frac{\text{Max}}{\text{Min}}$ response when $t = \frac{n\pi}{\omega_d}$ $\xrightarrow[\text{max }(n=1)]{\text{first}}$ $t_{max} = \frac{\pi}{\omega_d} = \frac{T_d}{2}$

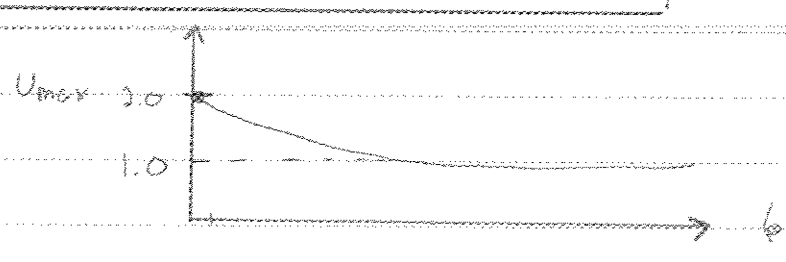
Evaluate $U(t)$ @ $t = t_{max}$ $\begin{cases}\cos\omega_d t_{max} = -1\\ \sin\omega_d t_{max} = 0\end{cases}$

$$u_{max} = \frac{P_o}{K}\left[1 + e^{-\xi\omega_n \pi/\omega_d}\right]$$

$$\boxed{U_{max} = \frac{P_o}{K}\left[1 + e^{-\pi\xi/\sqrt{1-\xi^2}}\right]}$$

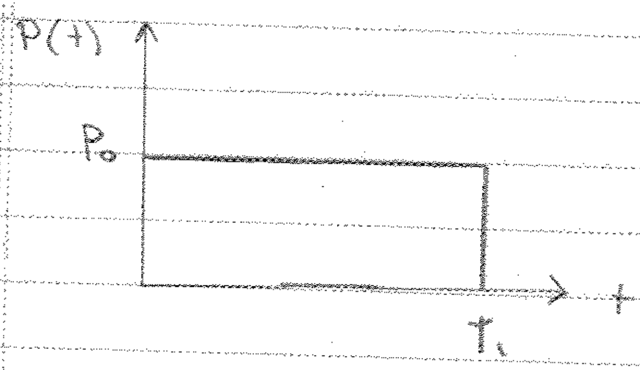
← (Also OK when $t_1 \geq \frac{T_d}{2}$)

ξ	U_{max}
0	2 P_o/K
.02	1.94 P_o/K
.10	1.73 P_o/K
∞	1



b) Effect of Blast Length –

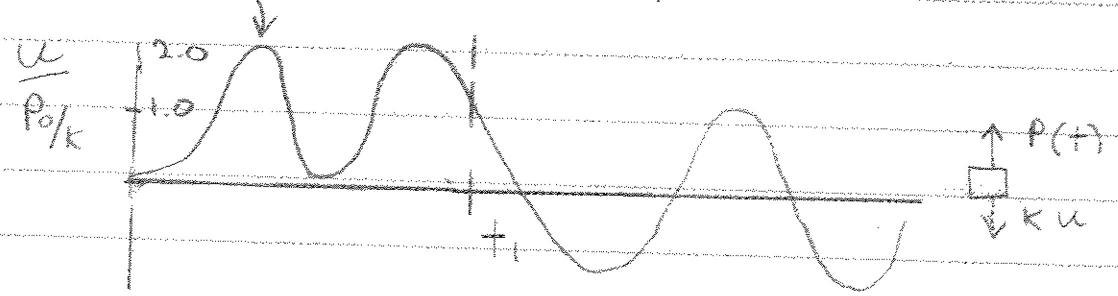
Could solve entire problem



$$\begin{cases} t < t_1 & P_o \\ t \geq t_1 & \text{Free Vibration} \end{cases}$$

Solve for Special Case: $\zeta = 0$

$$u(t) = \frac{P_o}{k}(1 - \cos \omega_n t)$$



Work $= \Delta T$:

$$\text{Work} = \int_0^{u_{max}} (P - ku)\, du = \Delta T = 0$$

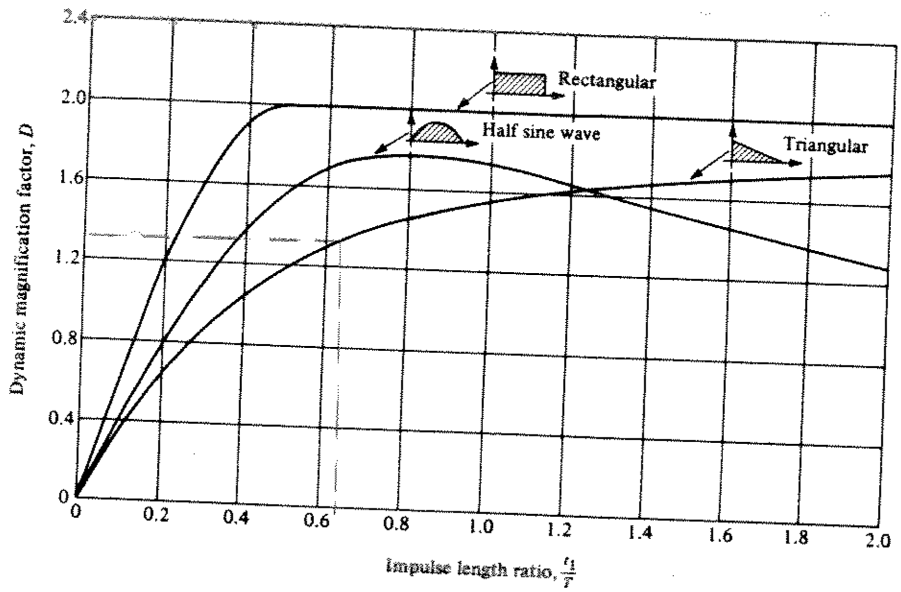
$$\int_0^{u_{t_1}} P\, du - \tfrac{1}{2} k u_{max}^2 = 0$$

$$P_o \underbrace{\left[\frac{P_o}{k}(1 - \cos \omega_n t_1)\right]}_{u(t_1)} = \tfrac{1}{2} k u_{max}^2$$

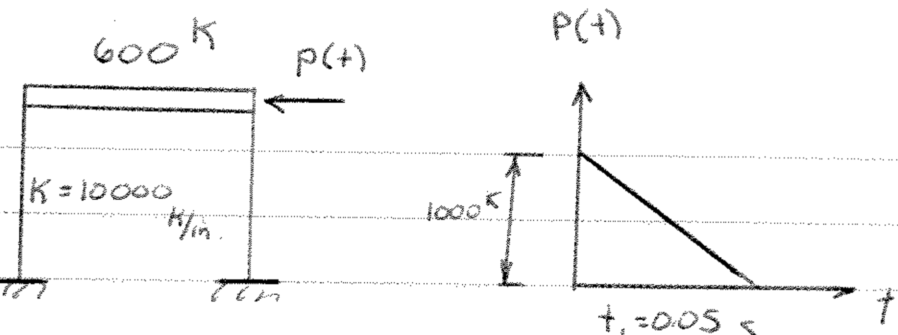
$$\boxed{u_{max} = \frac{P_o}{k}\sqrt{2\left(1 - \cos 2\pi \frac{t_1}{T_n}\right)}}$$

t_1/T_n	$\frac{u_{max}}{(P_o/k)}$
0	0
0.2	1.18
0.4	1.90
0.5	2.00

Blast Response Spectra



c) Example –



$T_n = 2\pi/\omega_n = 2\pi\sqrt{\dfrac{W}{gK}} = 2\pi\sqrt{\dfrac{600\ K}{10000\ K/in\ (386\ in/s^2)}} = 0.079\ s$

$\dfrac{t_1}{T_n} = \dfrac{0.05}{0.079} = 0.63$

From Spectrum $D = 1.33$

$U_{max} = D\ P_o/K = 1.33\left(\dfrac{1000\ K}{10000\ K/in}\right) = 0.133\ in$

$F_{max} = K\,U_{max} = 10{,}000\ (0.133) = \underline{\underline{1330\ \text{Kips}}}$