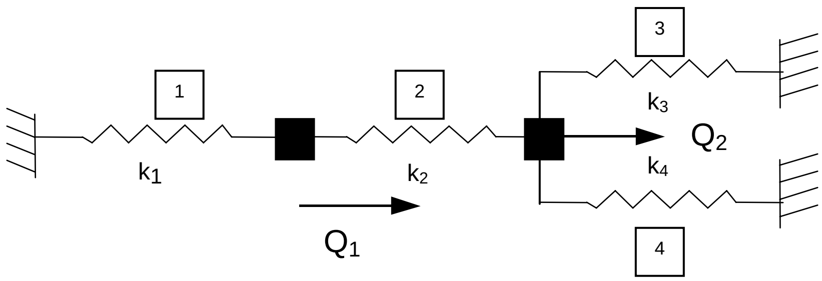
CEE 379 Problem Set 1 Autumn 2007

(Due 5:00 PM, Wednesday, Oct. 3 in 233 More)

Reading: Class notes

In all problems in this assignment, consider the four-spring, five-joint (node) system shown below. The displacements of three joints/nodes are known (displacements constrained to be 0.0), and the displacement of two joints/nodes are unknown (unconstrained). A force Q_1 is applied at the left free joint, and a force Q_2 is applied to the right free joint.



Problem 1. In solving this problem, **do not use matrices or spreadsheets.** Identify each answer by enclosing it in a box.

a) Only one independent equation can be written to describe the external equilibrium of a 1D-spring system. The degree of external indeterminacy is the number of additional independent equations (above those obtained from statics) necessary to solve for the external reactions. Based on the number of unknown reactions in this system (which you can determine from inspection), what is the degree of external static indeterminacy?

b) What is the degree of internal static indeterminacy?

c) What is the degree of kinematic indeterminacy (the number of unknown joint/node displacements)?

d) Draw free-body diagrams of all of the springs and nodes, annotating all spring end forces an reactions with the notation and sign convention discussed in class.

e) Based on these free-body diagrams, write the equations of equilibrium at each of the nodes. Number your nodes beginning with the unconstrained nodes, followed by the constrained nodes.

f) Write the end force-deformation relationships for all the springs in terms of the end displacements, end forces and spring stiffnesses of each spring.

g) By substituting the force-deformation relationships into the equations of equilibrium and by enforcing compatibility, write the equations of equilibrium at all of the nodes in terms of the spring stiffnesses, the joint displacements, and the applied forces and reactions.

Problem 2. In this problem and the next, assume that k_1 = 400 kN/m, k_2 = 800 kN/m, k_3 = 100 kN/m and k_4 = 350 kN/m. Assume that the left, free node is subjected to a 10 kN load to the right, and the right free node is subjected to a 22 kN load to the left. As in Problem 1, do not use matrices or spreadsheets in solving this problem.

a) By solving the first two equations from Problem 1, part g, determine the displacements at the free nodes.

b) By solving the last three equations from Problem 1, part g, determine the reactions at the constrained (fixed) nodes.

c) Considering the spring force-deformation relationships, solve for the end forces for all of the springs.

d) What are the internal spring forces T_1, T_2, T_3 and T_4? Show these internal spring forces on a neat sketch of the structure. Tension forces should be shown as positive and compressive forces shown as negative.

Problem 3. In this problem, you will repeat many of the previous calculations, but this time, you will use matrix notation.

a) Determine the spring stiffness matrices $\mathbf{k_1}$**,** $\mathbf{k_2}$**,** $\mathbf{k_3}$ **and** $\mathbf{k_4}$.

b) Determine the equilibrium equations in matrix form (i.e., **K D** = **Q**) for this spring system. Which components of **D** are unknown? Which components of **Q** are unknown?

c) Considering only the two equations (first two rows), which describes equilibrium at the free nodes, solve for the unknown displacements at the free joint. Compare your answer with the answer in Problem 2, part a.

d) Using the spring stiffness matrices, compute the end spring forces (i.e., **q** = **kd**) for all springs. Compare your answers with the answer in Problem 2, part c.

e) Compare the joint displacement and internal spring forces you calculated with the results of the 2DOF spreadsheet example provided in the notes. Are the results consistent with each other? Explain.