Autumn 2007

CEE 379 **Midterm Exam** Name: SOLUTION

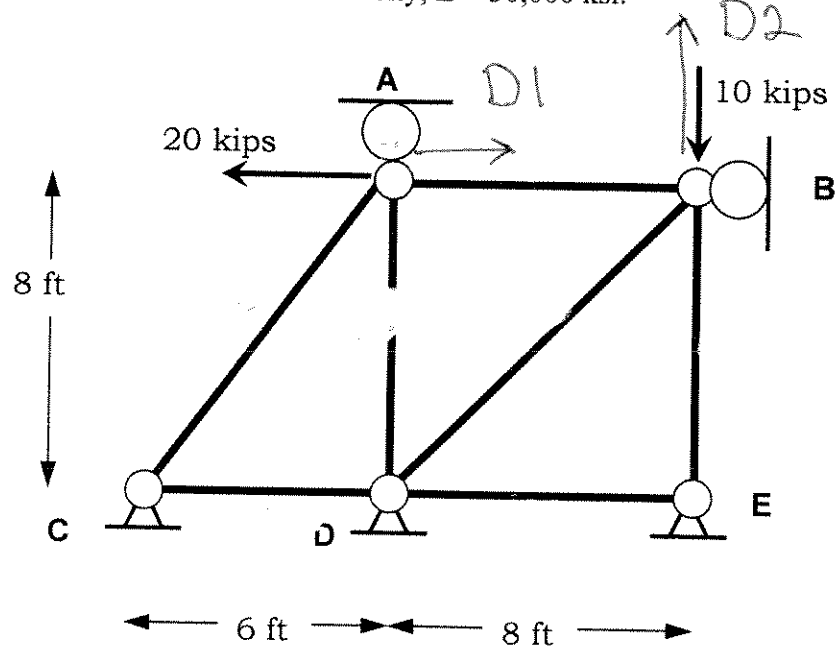
In solving this exam, you are permitted to consult two pages of hand-written notes (both sides), which you have prepared. Turn in these exam questions along with your solution.

10
25
30
35

SHOW ALL WORK

PROBLEM 1 (35 pts)

Consider the truss shown below. Joints C, D and E are fixed against translation. Joint A is fixed in the vertical direction but free to move horizontally. Joint B is fixed in the horizontal direction but free to move vertically. For all members, the cross-sectional area, A = 10 in^2, and the modulus of elasticity, E = 30,000 ksi.



a. (10) For this truss, determine the:

i) degree of external static indeterminacy 5

8 unknowns − 3 Eqn.

ii) degree of internal static indeterminacy 5

8 reactions + 7 members − (2)(5 joints)

iii) degree of kinematic indeterminacy 2

iv) dimensions of K_{11} matrix 2x2

v) dimensions of K_{22} matrix 8x8

b. (30) Joint A is subjected to a horizontal load of 20 kips, and Joint B is subjected to a vertical load of 10 kips, oriented as shown above. For this loading, determine the displacements of ALL joints in the horizontal and vertical directions using the direct stiffness method presented in class.

$$k_{CA} = \begin{bmatrix} & & \sim & \\ & & \sim & \\ & & 0.36 & \\ & & \sim & \end{bmatrix} \left(\frac{AE}{L}\right)_{CA}$$

30,000 k/ft = 2500 k/in

$$k_{DA} = \begin{bmatrix} & & \sim & \\ & & \sim & \\ & & 0.0 & \\ & & \sim & \end{bmatrix} \frac{AE}{L}_{DA}$$

3125 k/in

$$k_{AB} = \begin{bmatrix} 1.0 & & -1.0 & \\ \sim & & \sim & \\ \sim & & \sim & \\ 0 & & 0 & \end{bmatrix} \frac{AE}{L}_{AB}$$

3125 k/in.

$$k_{DB} = \begin{bmatrix} & & & \sim \\ & & & \sim \\ & & & \sim \\ & & & \left(\frac{\sqrt{2}}{2}\right)^2 \end{bmatrix} \frac{AE}{L}_{DB}$$

$= 1/2$

31824 k/ft

2210 k/in.

$$k_{EB} = \begin{bmatrix} & & & \sim \\ & & & \sim \\ & & & \sim \\ & & & 1.0 \end{bmatrix} \frac{AE}{L}_{EB}$$

37500 k/ft

3125 k/in.

$\underbrace{0.36(2500)}_{900} + 3125 + 0$

$$\Longrightarrow [K_{11}] = \begin{bmatrix} 4025 & 0 \\ 0 & 4230 \end{bmatrix} \frac{\text{kip}}{\text{in.}}$$

$0 + 3125 + \frac{1}{2}(2210)$

$$\begin{bmatrix} -20 \\ -10 \end{bmatrix} \text{kip} = [K_{11}] \cdot [D_U] \longrightarrow \boxed{D_U = \begin{bmatrix} -0.0050 \\ -0.0024 \end{bmatrix} \text{in}}$$

(DOF 1)

(DOF)

(+)

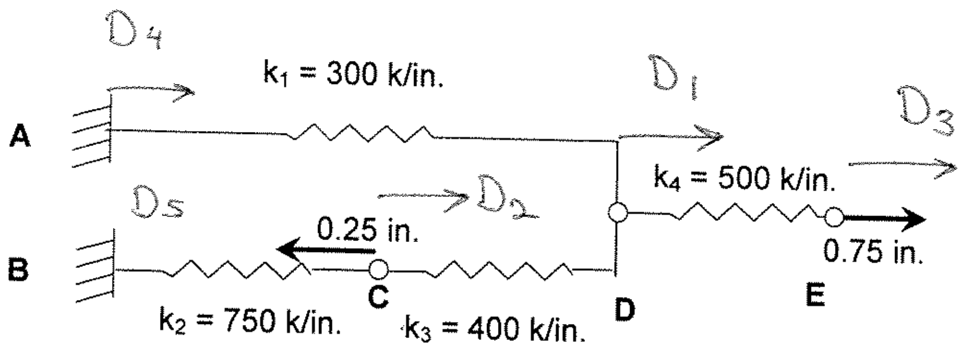
(+)

$$= \begin{bmatrix} -0.000414 \\ -0.000197 \end{bmatrix} \text{ft.}$$

All other displacements = 0.0

PROBLEM 2 (30 pts)

Consider the 1D spring system shown below. Assume that the joints move only in the horizontal direction and that the joints do not rotate. Joint C is subjected to a horizontal displacement of 0.25 inches to the left. Joint E is subjected to a 0.75-inch displacement to the right.



a. (25) Using the direct stiffness method, determine the displacements of all the joints.

$$k_1 = \begin{array}{cc} \text{④} & \text{①} \\ \end{array} \begin{bmatrix} 300 & -300 \\ -300 & 300 \end{bmatrix} \begin{array}{c} \text{④} \\ \text{①} \end{array} \quad k_2 = \begin{array}{cc} \text{⑤} & \text{②} \end{array} \begin{bmatrix} 750 & -750 \\ -750 & 750 \end{bmatrix} \begin{array}{c} \text{⑤} \\ \text{②} \end{array} \quad k_3 = \begin{array}{cc} \text{②} & \text{①} \end{array} \begin{bmatrix} 400 & -400 \\ -400 & 400 \end{bmatrix} \begin{array}{c} \text{②} \\ \text{①} \end{array} \quad k_4 = \begin{array}{cc} \text{①} & \text{③} \end{array} \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{array}{c} \text{①} \\ \text{③} \end{array}$$

$$Q = \begin{array}{c} Q_k \\ Q_u \end{array} \begin{bmatrix} 0 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix} = \begin{bmatrix} \begin{matrix} 300 \\ 400 \\ 500 \end{matrix} & -400 & -500 & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} \begin{Bmatrix} D_1 \\ -0.25 \\ 0.75 \\ 0 \\ 0 \end{Bmatrix} \begin{array}{c} D_u \\ D_K \end{array}$$

$$0 = (300 + 400 + 500) D_1 - 400(-0.25) - 500(0.75)$$

$$D_1 = \frac{-100 + 375}{1200} = 0.229 \text{ in.}$$

$$D = \begin{Bmatrix} 0.23 \\ -0.25 \\ 0.75 \\ 0 \\ 0 \end{Bmatrix}$$

b. (5) Determine the axial force in spring CD. Take tension as positive and compression as negative.

$$q_{CD} = \begin{bmatrix} 400 & -400 \\ -400 & 400 \end{bmatrix} \begin{Bmatrix} -0.25 \\ 0.229 \end{Bmatrix} = \begin{Bmatrix} -191.7 \\ +191.7 \end{Bmatrix} K$$

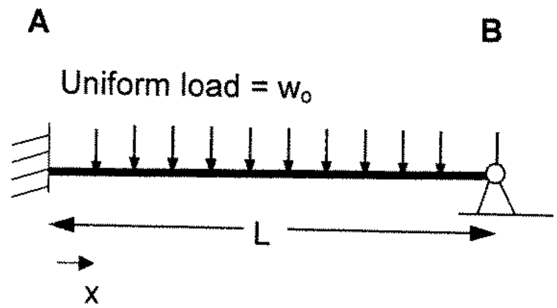
$$\xleftarrow{\quad} \overset{191.7^{K}}{\overline{\qquad\qquad}} \xrightarrow{\quad}$$

$$\boxed{\text{Tension} = 191.7 \text{ K}}$$

PROBLEM 3 (35 pts)

Consider the beam shown below, which is fixed on one end and pinned at the other. It has length, L, moment of inertia, I and elastic modulus, E. It is subjected to a downward uniform load of magnitude w_o.

a. (23) By direct integration of the beam equation, determine the deflection (displacement) of the beam as a function of the distance along the beam, x.



$$v''''(x) = -w_o/EI$$

$$v'''(x) = -\frac{w_o}{EI}x + C_1$$

$$v''(x) = -\frac{w_o}{EI}\frac{x^2}{2} + C_1 x + C_2$$

$$v'(x) = -\frac{w_o}{EI}\frac{x^3}{6} + C_1\frac{x^2}{2} + C_2 x + C_3$$

$$v(x) = -\frac{w_o}{EI}\frac{x^4}{24} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$

$$v(0) = \boxed{0 = C_4}$$

$$v'(0) = \boxed{0 = C_3}$$

$$EI\,v(L) = 0 = -\frac{w_o L^4}{EI\ 24} + C_1\frac{L^3}{6} + C_2\frac{L^2}{2}$$

$$EI\,v''(L) = 0 = \left(-w_o\frac{L^2}{2} + C_1 L + C_2\right)EI \quad \text{multiply by } \left(-\frac{L^2}{2EI}\right)$$

$$-w_o L^4\left(\frac{1}{24} - \frac{1}{4}\right) + C_1 L^3\left(1/6 - 1/2\right) = 0$$

$$+\frac{5}{24}w_o L^4 - 1/3\, C_1 L^3 = 0$$

$$C_1 = +\frac{15}{24}\frac{w_o L}{EI} = \frac{5}{8}\frac{w_o L}{EI} \qquad C_2 = -\frac{1}{8}\frac{w_o L^2}{EI}$$

$$\boxed{v(x) = -\frac{w_o x^4}{24EI} + \frac{w_o L}{EI}\left(\frac{5}{48}\right)x^3 - \frac{w_o L^2}{16EI}x^2}$$

$$v(x) = \frac{w_o L^4}{48EI}\left[\left(-2\left(\frac{x}{L}\right)^4 + 5\left(\frac{x}{L}\right)^3 - 3\left(\frac{x}{L}\right)^2\right]$$

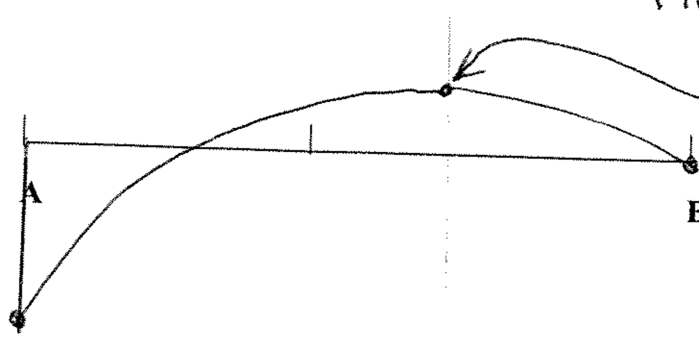
b. (5) On the drawing below, plot the **bending moment diagram** for the beam. On the plot, indicate the magnitude of the maximum positive and negative bending moments.

$$M(x) = EI\,v''(x) = \left(-\frac{w_o}{EI}\frac{x^2}{2} + C_1 x + C_2\right)EI$$

$$= -w_o\frac{x^2}{2} + \frac{15}{24}w_o L x - \frac{1}{8}w_o L^2$$

$$M_{max} = \tfrac{1}{2}\left(\frac{3L}{8}\right)\left(\frac{3}{8}w_o L\right) = \frac{9}{128}w_o L^2$$

Bending Moment Diagram



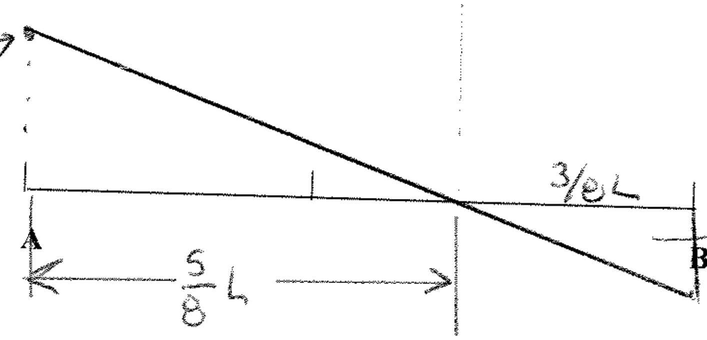
$M(0) = -\frac{1}{8}w_o L^2$

c. (5) On the drawing below, plot the **shear force diagram** for the structure. Indicate the magnitude of the maximum and minimum shear forces.

$$V(x) = EI\,v'''(x) = \left(-\frac{w_o}{EI}x + C_1\right)EI$$

$$= -w_o x + \frac{15}{24}w_o L = -w_o x + \frac{5}{8}w_o L$$

Shear Force Diagram



$V(0) = \frac{5}{8}w_o L$

$V(L) = -w_o L + \frac{5}{8}w_o L = -\frac{3}{8}w_o L$

d. (2) Using the sign convention for the direct stiffness method, what is the **end moment** at point A? Show all work.

$M(0) = -\frac{w_o L^2}{8}$ ↶ −

$$\boxed{M_A = +\frac{w_o L^2}{8}}$$