



Spring Force-Deformation Relationships

	$\{q\}$	=	$[k']$	*	$\{d\}$
Spring 1 $k_1 = 500 \text{ kN/m}$	$\begin{bmatrix} q_N \\ q_F \end{bmatrix}$	=	$\begin{matrix} & 3 & 1 \\ \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} & & \end{matrix}$	*	$\begin{bmatrix} d_N \\ d_F \end{bmatrix} \begin{matrix} 3 \\ 1 \end{matrix}$
Spring 2 $k_2 = 1000 \text{ kN/m}$	$\begin{bmatrix} q_N \\ q_F \end{bmatrix}$	=	$\begin{matrix} & 1 & 2 \\ \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} & & \end{matrix}$	*	$\begin{bmatrix} d_N \\ d_F \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$
Spring 3 $k_3 = 250 \text{ kN/m}$	$\begin{bmatrix} q_N \\ q_F \end{bmatrix}$	=	$\begin{matrix} & 2 & 4 \\ \begin{bmatrix} 250 & -250 \\ -250 & 250 \end{bmatrix} & & \end{matrix}$	*	$\begin{bmatrix} d_N \\ d_F \end{bmatrix} \begin{matrix} 2 \\ 4 \end{matrix}$

Compatibility

Boundary	$d_{N1} = D_3 = 0$	Nodal	$d_{F1} = d_{N2} = D_1$
Conditions:	$d_{F3} = D_4 = 0$	Connectivity:	$d_{F2} = d_{N3} = D_2$

System Stiffness Equations

$[K]$				*	$\{D\}$	=	$\{Q\}$	
1	2	3	4					
1500	-1000	-500	0	1	*	D_1	=	Q_1
-1000	1250	0	-250	2		D_2		Q_2
-500	0	500	0	3		D_3		Q_3
0	-250	0	250	4		D_4		Q_4

Solve for Nodal Displacements

$$\begin{array}{c}
 \mathbf{[K_{11}]} \\
 \begin{array}{cc}
 1 & 2 \\
 \boxed{\begin{array}{cc} 1500 & -1000 \\ -1000 & 1250 \end{array}} \\
 \begin{array}{c} 1 \\ 2 \end{array}
 \end{array}
 * \mathbf{\{D_u\}} = \mathbf{\{Q_k\}}
 \end{array}$$

$$\begin{array}{c}
 \mathbf{[K_{11}^{-1}]} \\
 \boxed{\begin{array}{cc} 0.0014 & 0.0011 \\ 0.0011 & 0.0017 \end{array}}
 * \mathbf{\{Q_k\}} = \mathbf{\{D_u\}}
 \end{array}$$

$$\begin{array}{c}
 * \mathbf{\begin{array}{c} \boxed{D_1} \\ \boxed{D_2} \end{array}} = \mathbf{\begin{array}{c} \boxed{Q_1} \\ \boxed{Q_2} \end{array}} \\
 * \mathbf{\begin{array}{c} \boxed{10} \\ \boxed{-25} \end{array}} = \mathbf{\begin{array}{c} \boxed{-0.014} \text{ m} \\ \boxed{-0.031} \text{ m} \end{array}}
 \end{array}$$

Spring End Forces

(positive to right, negative to left)

$$\begin{array}{c}
 \mathbf{[k']} \\
 \begin{array}{cc}
 \text{Spring 1} & \boxed{\begin{array}{cc} 500 & -500 \\ -500 & 500 \end{array}} \\
 \text{Spring 2} & \boxed{\begin{array}{cc} 1000 & -1000 \\ -1000 & 1000 \end{array}} \\
 \text{Spring 3} & \boxed{\begin{array}{cc} 250 & -250 \\ -250 & 250 \end{array}}
 \end{array}
 * \mathbf{\{d\}} = \mathbf{\{q\}}
 \end{array}$$

$$\begin{array}{c}
 * \mathbf{\begin{array}{c} \boxed{0.000} \\ \boxed{-0.014} \end{array}} = \mathbf{\begin{array}{c} \boxed{7.14} \text{ kN} \\ \boxed{-7.14} \text{ kN} \end{array}} \\
 * \mathbf{\begin{array}{c} \boxed{-0.014} \\ \boxed{-0.031} \end{array}} = \mathbf{\begin{array}{c} \boxed{17.14} \text{ kN} \\ \boxed{-17.14} \text{ kN} \end{array}} \\
 * \mathbf{\begin{array}{c} \boxed{-0.031} \\ \boxed{0.000} \end{array}} = \mathbf{\begin{array}{c} \boxed{-7.86} \text{ kN} \\ \boxed{7.86} \text{ kN} \end{array}}
 \end{array}$$

Spring Internal Forces

(tension positive, compression negative)

$$\begin{array}{c}
 \text{Spring 1} \quad T_1 = \quad -7.14 \text{ kN} \\
 \text{Spring 2} \quad T_2 = \quad -17.14 \text{ kN} \\
 \text{Spring 3} \quad T_3 = \quad 7.86 \text{ kN}
 \end{array}$$