Homework 8; Due 8.30 a.m. Wed Mar 4. Ross Problems: Ch 6; 2(a)&35, 3(a)&37, 28; Ch 7 48, 51.

Ch 6: 2(a)&35; Urn of 13 balls, of which 5 are white; 3 balls chosen.

2(a) $X_i = 1$ if ith ball selected is white;

$P(X_1 = 1) \;=\; 5/13.$ $P(X_2 = 1 \mid X_1 = 1) = 4/12$, $P(X_2 = 0 \mid X_1 = 1) \;=\; 8/12$

So $P((X_1, X_2) = (1, 1)) = 5/39$, $P((X_1, X_2) = (1, 0)) \;=\; P((X_1, X_2) = (0, 1)) = 10/39$,

and $P((X_1, X_2) = (0, 0)) = (8/13)(7/12) \;=\; 14/39$.

35(a) Note $P(X_2 = 1) \;=\; 5/13$;

$P(X_1 = 1 \mid X_2 = 1) \;=\; (5/39)/(5/13) \;=\; 1/3$ $P(X_1 = 0 \mid X_2 = 1) \;=\; 1 - (1/3) \;=\; 2/3$.

35(b) $P(X_2 = 0) = 1 - 5/13 \;= \;= 8/13$.

$P(X_1 = 1 \mid X_2 = 0) \;=\; (10/39)/(8/13) \;=\; 5/12$, $P(X_1 = 0 \mid X_2 = 0) \;=\; 1 - (5/12) \;=\; 7/12$.

Ch 6: 3(a)&37; Urn of 13 balls, (of which 5 are white); 3 balls chosen.

$Y_i = 1$ if ith white ball is selected; $P(Y_i = 1) \;=\; 3/13.$ $P(Y_2 = 1 \mid Y_1 = 1) = 2/12$,

So $P((Y_1, Y_2) = (1, 1)) = 1/26$, $P((Y_1, Y_2) = (1, 0)) \;=\; P((Y_1, Y_2) = (0, 1)) = 5/26$,

and $P((Y_1, Y_2) = (0, 0)) = (26 - 1 - 5 - 5)/26 = 15/26$.

37(a) Note $P(Y_2 = 1) \;=\; 3/13$;

$P(Y_1 = 1 \mid Y_2 = 1) \;=\; (1/26)/(3/13) \;=\; 1/6$ $P(Y_1 = 0 \mid Y_2 = 1) \;=\; 1 - (1/6) \;=\; 5/6$.

37(b) $P(Y_2 = 0) = 1 - 3/13 \;=\; 10/13$;

$P(Y_1 = 1 \mid Y_2 = 0) \;=\; (5/26)/(10/13) \;=\; 1/4$, $P(Y_1 = 0 \mid Y_2 = 0) \;=\; 1 - (1/4) \;=\; 3/4$.

Ch 6 28; $Z = X_1/X_2$, $X_1 \;\sim\; \mathcal{E}(\lambda_1)$, $X_2 \;\sim\; \mathcal{E}(\lambda_2)$, X_1 and X_2 independent.

$$\begin{aligned} F_Z(z) \;&=\; P(Z \le z) \;=\; P(X_1 \le zX_2) \;=\; \int_{y=0}^{\infty}\int_{x=0}^{zy} f_{X_1}(x) f_{X_2}(y)\; dx\; dy \;=\; \int_{y=0}^{\infty} F_{X_1}(zy) f_{X_2}(y)\; dy \\ &=\; \int_{y=0}^{\infty} (1 - \exp(-\lambda_1 zy))\lambda_2 \exp(-\lambda_2 y)\; dy \;=\; 1 \;-\; \lambda_2/(\lambda_1 z + \lambda_2) \;=\; \lambda_1 z/(\lambda_1 z + \lambda_2) \end{aligned}$$

So $F_Z(z) \;=\;$ on $z \ge 0$ and $F(Z_z) \;=\; 0$ for $z < 0$.

Also $P(X_1 < X_2) \;=\; P(Z < 1) \;=\; F_Z(1) \;=\; \lambda_1/(\lambda_1 + \lambda_2)$.

Ch 7 48. X is number of tries to get a 6; Y is number of rolls to get a 5.

(a) $X \;\sim\; Geo(1/6)$; $P(X = k) \;=\; 5^{k-1}/6^k$, $k = 1, 2, \ldots$, $\mathrm{E}(X) = 6$.

(b) Given $Y = 1$, $X \;\sim\; 1 + Geo(1/6)$. $\mathrm{E}(X \mid Y = 1) = 1 + \mathrm{E}(X) \;=\; 7$.

(c) Given $Y = 5$, $X = 1, 2, 3, 4$ with probs $4^{k-1}/5^k$, and $P(X \le 4 \mid Y = 5) \;=\; 0.5904$

and $X = 5 + k$ with probs $(1 - 0.5904) * 5^{k-1}/6^k$,

giving $\mathrm{E}(X \mid Y = 5) \;=\; 1.3136 + 0.4896 * (5 + 6) \;=\; 5.8192$

Ch 7 51. $f_{X,Y}(x, y) \;=\; \exp(-y)/y$ on $0 < x < y < \infty$.

So $f_Y(y) \;=\; \int_{x=0}^{y} \exp(-y)/y\; dy \;=\; \exp(-y)$ on $0 < x < \infty$.

So $f_{X|Y}(x|y) \;=\; f_{X,Y}(x, y)/f_Y(y) \;=\; 1/y$ on $0 < x < y$.

So, given $Y = y$, X is $U(0, y)$, and $\mathrm{E}(X^3 \mid Y = y) \;=\; (1/y)\int_0^y x^3\; dx \;=\; y^3/4$.