Homework 4; Due 8.30 a.m. Wed Feb 4. Ross Problems: Ch 6: 1(a),(b); 6, 10, 20, 21

Ch 6: 1 (a)(b) Each die is 1,2,3,4,5,6 independently with prob 1/6.

(a) probabilities given in table: multiply by 1/36. Margins given as check.

For (a) note $X < Y \leq 2X$.

$X =$ max	$Y =$ sum 2	3	4	5	6	7	8	9	10	11	12	total
1	1											1
2		2	1									3
3			2	2	1							5
4				2	2	2	1					7
5					2	2	2	2	1			9
6						2	2	2	2	2	1	11
total	1	2	3	4	5	6	5	4	3	2	1	36

(b)

For (b), note $X \leq Y$.

$X =$ first	$Y =$ max 1	2	3	4	5	6	total
1	1	1	1	1	1	1	6
2		2	1	1	1	1	6
3			3	1	1	1	6
4				4	1	1	6
5					5	1	6
6						6	6
total	1	3	5	7	9	11	36

Ch 6: 6;

N_1	N_2 1	2	3	4	
1	0.1	0.1	0.1	0.1	0.4
2	0.1	0.1	0.1		0.3
3	0.1	0.1			0.2
4	0.1				0.1
total	0.4	0.3	0.2	0.1	1

Easiest is just to write down the 10 possibilities for the locations of the 2 bad ones in the 5. Each of these possibilities gives a unique (N_1, N_2) and has probability 1/10 or 0.1.

Ch 6: 10. $f_{X,Y}(x,y) = \exp(-x-y)$ on $0 \leq x < \infty$, $0 \leq y < \infty$.

(a) $P(X < Y) = 1/2$ by symmetry. Or:

$$P(X<Y) = \int\int_{x<y} \exp(-x-y)\ dx\ dy = \int_{x=0}^{\infty}\int_{y=x}^{\infty} \exp(-x)\exp(-y)\ dy\ dx = \int_x \exp(-2x)\ dx = (1/2).$$

(b) $P(X < a) = \int_{x=-\infty}^{a}\int_{y=-\infty}^{\infty} \exp(-x)\exp(-y)\ dx\ dy = \int_{x=-\infty}^{a} \exp(-x)\ dx = (1-\exp(-a))$.

Ch 6: 20.

(a) $f(x,y) = x\exp(-x-y)$ on $0 < x < \infty,\ 0 < y < \infty$,
so $f_X(x) = x\exp(-x)\int_0^\infty e^{-y}dy = x\exp(-x)$ on $x > 0$
and $f_Y(y) = \exp(-y)\int_0^\infty x\exp(-x)dx = \exp(-y)$ on $y > 0$.
i.e. $f(x,y) = f_X(x)f_Y(y)$ so X and Y are independent.

Or ok just to cite factorization, so long as make it clear that factorization is for all $-\infty < x < \infty$, $-\infty < y < \infty$.

(b) No, X and Y cannot be independent, since $X < Y$.
In fact $f_Y(y) = \int_0^y 2dx = 2y$ on $0 < y < 1$, so $f(x,y) \neq f_X(x)f_Y(y)$.

Ch 6: 21. (a)
$$\int\int_{x+y<1} f(x,y)\ dx\ dy = \int_{x=0}^{1}\int_{y=0}^{1-x} 24\ x\ y\ dy\ dx$$
$$= \int_{x=0}^{1} 12\ x\ (1-x)^2 dx = [6\ x^2\ -\ 8\ x^3\ +\ 3\ x^4]_0^1 = 1.$$

So $f(x,y) \geq 0$ and integrates to 1; hence it is a pdf.

(b) From (a): $f_X(x) = 12x(1-x)^2$ on $0 < x < 1$.

So $\mathrm{E}(X) = \int_0^1 12x^2(1-x)^2\ dx = [4x^3 - 6x^4 + 12x^5/5] = 2/5$..

(c) By symmetry: $\mathrm{E}(Y) = \mathrm{E}(X)$, so $\mathrm{E}(Y) = 2/5$.