Homework 3; Due 8.30 a.m. Wed Jan 28. Ross Problems: Ch 5: 16, 19, 26 Ch 5: TE 9, TE 12

Ch 5, 16; Annual rainfall X is $N(40, 16)$. $P(X > 50) \;=\; P(Z \equiv (X-40)/4 \;>\; 2.5) = 1 - 0.9938 = 0.0062$. Assuming years are independent (state this assumption), probability it is more than 10 years is probability next 10 years all have < 50 inches, or $(0.9938)^{10} = 0.94$.

Ch 5, 19; X is $N(12, 4)$, so $Z \equiv (X - 12)/2$ is $N(0, 1)$.
$X > c$ is $Z > (c-12)/2)$, and from table $P(Z > 1.28) = 0.1$, which corresponds to $c = 12+2\times 1.28 \;=\; 14.56$.

Ch 5, 26; If coin is fair, $P(H) = 0.5$. Otherwise $P(H) = 0.55$.
If coin is fair, we will wrongly conclude biased if 525 or more heads in 1000 tosses. $Z = (X-500)/\sqrt{1000 \times 0.5 \times 0.5}$ is approx $N(0, 1)$, and we want probability $X > 524.5$ or $Z = (X - 500)/\sqrt{1000 \times 0.5 \times 0.5} \;>\; 1.55$. From the table this is $1 - 0.94 \;=\; 0.06$.
If coin is biased, we will wrongly conclude fair if 524 of less heads in 1000 tosses. $Z = (X-550)/\sqrt{1000 \times 0.55 \times 0.45}$ is approx $N(0, 1)$, and we want probability $X < 524.5$ or $Z = (X - 550)/\sqrt{1000 \times 0.55 \times 0.45} \;<\; -1.627$. From the table this is $P(Z > 1.627) \;=\; 1 - 0.948 \;=\; 0.052$.

Ch 5: TE 9; (a) Let $f()$ denote the standard Normal density, and note $f(z) = f(-z)$. Let $w = -z$.

$$P(Z > x) \;=\; \int_x^{\infty} f(z)\; dz \;=\; -\int_{-\infty}^{-x} f(-w)\;(-dw) \;=\; \int_{-\infty}^{-x} f(w)\; dw \;=\; P(Z < -x)$$

(b)

$$P(|Z| > x) \;=\; P(Z > x) \;+\; P(Z < -x) \;=\; 2P(Z > x) \quad \text{from (a).}$$

(c)

$$P(|Z| < x) \;=\; 1 \;-\; P(|Z| > x) \;=\; 1 \;-\; 2P(Z > x) \;=\; 1 - 2(1 - P(Z < x)) \;=\; 2P(Z < x) - 1$$

Ch 5: TE 12; (a) X is $U(a, b)$, $F_X(x) \;=\; P(X \leq x) \;=\; (x - a)/(b - a)$.
If $F_X(m) = 1/2$, $2(m - a) = (b - a)$, so $m = a + (b - a)/2 = (a + b)/2$.
(b) X is $N(\mu, \sigma^2)$. The density is symmetric about μ.
So by symmetry, $F_X(\mu) = 1/2$, and $m = \mu$.
(c) X is $\mathcal{E}(\lambda)$, so $F_X(x) = 1 - \exp(-\lambda x)$.
So, $F_X(m) = 1/2$ gives $\exp(-\lambda\; m) = 1/2$, $m \;=\; -\log(1/2)/\lambda \;=\; 0.69/\lambda$.