Homework 1; Due 8.30 a.m. Wed Jan 14: Stat394 Final Exam

1. (a) Let T be the event of testing positive.

$$P(T) = P(T \mid G)P(G) + P(T \mid G^c)P(G^c) = 0.95 \times .01 + 0.05 \times 0.99 = 0.059$$

(b) $P(G \mid T) = P(T \mid G)P(G)/P(T) = 0.95 \times .01/0.059 = 0.161,$

$$P(G^c \mid T) = 1 - P(G \mid T) = 1 - 0.161 = 0.839. \text{ (Or, } .99 \times 0.05/0.059 = 0.839)$$

- (c) $P(G \mid T^c) = P(T^c \mid G)P(G)/P(T^c) = .01 \times .05/(1 0.059) = .00053.$
- 2 (a) Note having H and testing positive for H are same thing, since there is no error in testing for H. $P(H \mid T) = P(H \mid G)P(G \mid T) + P(H \mid G^c)P(G^c \mid T) = .161 \times (3/4) + .839 \times (1/4) = .3305$

(b)
$$P(H \cap G^c \cap T) = P(H \mid (G^c \cap T))P(G^c \cap T) = P(H \mid G^c)P(G^c \mid T)P(T) = .25 \times .839 \times .059 = 0.0124$$

Or,
$$P(H \cap G^c \cap T) = P(H \mid (G^c \cap T))P(G^c \cap T) = P(H \mid G^c)P(T \mid G^c)P(G^c) = .25 \times 0,05 \times .99 = 0.0124.$$

- (c) $P(H \cap G \cap T^c) = P(H \mid G)P(G \mid T^c)P(T^c) = 0.75 \times .00053 \times (1 0.059) = 0.000375.$
- Or $P(H \cap G \cap T^c) = P(H \mid G)P(T^c \mid G)P(G) = 0.75 \times 0.05 \times .01 = 0.000375.$
- 3. (a) $P(\text{bad}) = 1 P(\text{good}) = 1 P(0 \text{ faulty}) P(1 \text{ faulty}) = 1 (0.9)^{10} 10 \times 0.1 \times 0.9^9 = 0.264$
- (b) $P(2^{nd} \text{ bad is on trial } 9) = {8 \choose 1} \times .1^2 \times .9^7 = 0.0383.$ $P(\text{first } 9 \text{ all good}) = 0.9^9 = 0.387.$

 $P(\text{sold} \mid \text{exactly 9 tested}) = 0.387/(0.387 + 0.0383) = 0.91.$

- (c) $P(\text{gadget sold} \mid \text{bad}) = P(\text{sold})P(\text{bad} \mid \text{sold})/P(\text{bad}) =$ $P(\text{first 5 good}) \times P(2 \text{ or more bad in 2nd 5})/P(\text{bad}) = .59 \times .08/0.264 = 0.1788$ $P(\text{gadget discarded} \mid \text{good}) = P(1 \text{ bad in first 5}) \times P(0 \text{ in untested 5})/P(\text{good})$
 - $= .33 \times .59/(1-0.264) = 0.2645$. (Note if more than 1 bad in first 5, gadget cannot be good)
- 4. (a) Let T be number of winners in any week. T is approx Poisson with mean E(T)=40000/20000=2. P(T>2)=1-P(T=0)-P(T=1)-P(T=2)=

$$1 - \exp(-2) - 2\exp(-2) - 2^2 \exp(-2)/2 = 1 - 5\exp(-2) = .323.$$

- (b) Let X be the number of weeks (out of 208) with more than 2 winners. (Note $X \sim Bin(208, 0.323)$.) $E(X) = 208 \times 0.323 = 67.25$, $var(X) = 208 \times 0.323 \times (1 0.323) = 45.5$.
- (c) Approximately a standard Normal is $Z = (X 67.25)/\sqrt{45.5}$ and we want P(X > 59.5) = P(Z > -1.15). From Table P(Z < 1.15) = 0.875 and, by symmetry, this is the required probability.
- 5. (a) $P(T > t) = \int_t^\infty f_T(s)ds = \int_t^\infty \lambda \exp(-\lambda s)ds = [-\exp(-\lambda s)]_t^\infty = \exp(-\lambda t)$
- (b) Let the failure times be T_1 and T_2 : $P(T_1 > t) = P(T_2 > t) = \exp(-t)$ (from (a)).

By independence: $P(T_1 > t \cap T_2 > t) = \exp(-t) \times \exp(-t) = \exp(-2t)$.

Hence from (a), with $\lambda = 2$, (or by differentiating) we have the required pdf $2 \exp(-2t)$.

- (c) Suppose first component fails at s:
- we want $P(T_2 > t + s \mid T_2 > s) = P(T_2 > (t + s))/P(T_2 > s) = \exp(-(t + s))/\exp(-s) = \exp(-t)$.

(or, one can cite forgetting property of exponential here.)

Hence from (a), or by differentiating we get the required pdf $\exp(-t)$ on $0 < t < \infty$.

- (d) After each repair, it runs in full mode for, on average, 0.5 weeks, and then as a copier for, on average, 1 week. So between each repair, it runs on average 1.5 weeks, and for 0.5 weeks in full mode: 0.5/1.5 = 1/3.
- OR: rate of failing when in full mode is twice the rate when running only as a copier,

(Note this is NOT a proof, just a reasonable explanation.)