MATH/STAT 395 Midterm-1: Winter 2009 Feb 6, 2009
Useful facts

1. Permutations and combinations

There are $n! = \prod_{i=1}^{n} i = 1.2.3.4.\ldots.n$ permutations of n objects.

There are $\binom{n}{k} = n!/(k!(n-k)!)$ ways of choosing a given k objects from n.

2. Joint and conditional probabilities

If C and D are any events: $\mathrm{P}(C \bigcup D) = \mathrm{P}(C) + \mathrm{P}(D) - \mathrm{P}(C \bigcap D)$.

The conditional probability of C given D is $\mathrm{P}(C \mid D) = \mathrm{P}(C \bigcap D) / \mathrm{P}(D)$.

C and D are independent if $\mathrm{P}(C \bigcap D) = \mathrm{P}(C).\mathrm{P}(D)$.

3. Laws and theorems

Suppose $E_1, \ldots, E_k$ is a partition of Ω.

That is $E_i \bigcap E_j$ is empty, and $E_1 \bigcup E_2 \bigcup \ldots \bigcup E_k = \Omega$.

The law of total probability states that: $\mathrm{P}(D) = \sum_{j=1}^{k} \mathrm{P}(D \bigcap E_j) = \sum_{j=1}^{k} \mathrm{P}(D \mid E_j)\, \mathrm{P}(E_j)$

Bayes' Theorem states that: $\mathrm{P}(E_i \mid D) = \mathrm{P}(D \mid E_i)\, \mathrm{P}(E_i)/\mathrm{P}(D)$

4. Random variables and distributions

	discrete (mass)	continuous (density)
Probability mass/density function	pmf: $\mathrm{P}(X = x) = p_X(x)$	pdf: $f_X(x)$
Cumulative dist. func. CDF, $\mathrm{P}(X \le x)$	$F_X(x) = \sum_{y \le x} p_X(y)$	$F_X(x) = \int_{-\infty}^{x} f_X(y)dy$
Joint mass/density func. of (X, Y)	$p_{X,Y}(x, y) = \mathrm{P}(X = x, Y = y)$	$f_{X,Y}(x, y)$
Marginal mass/density of X	$p_X(x) = \sum_y \mathrm{P}(X = x, Y = y)$	$f_X(x) = \int_{y=-\infty}^{\infty} f_{X,Y}(x, y)dy$
Independence of X and Y	$p_{X,Y}(x, y) = \mathrm{P}(X = x)\mathrm{P}(Y = y)$	$f_{X,Y}(x, y) = f_X(x)f_Y(y)$

Expectation: $\mathrm{E}(X) = \sum_x x\ \mathrm{P}(X = x)$ or $\int_{-\infty}^{\infty} x f_X(x)dx$ and

$\mathrm{E}(g(X)) = \sum_x g(x)\ \mathrm{P}(X = x)$ or $\int_{-\infty}^{\infty} g(x) f_X(x)dx$ **provided** the sum/integral converges.

Variance: $\mathrm{var}(X) = \mathrm{E}((X - E(X))^2) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

5. Standard distributions:

	pmf or pdf	mean	variance
(a) Binomial; $B(n, p)$	$\mathrm{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$	np	$np(1-p)$
index n, parameter p	$k = 0, 1, 2, ..., n$		
(b) Geometric; $G(p)$;	$\mathrm{P}(X = k) = p(1-p)^{k-1}$	$1/p$	$(1-p)/p^2$
parameter p	$k = 1, 2, 3, 4, ...$		
(c) Neg. Binomial; $NegB(r, p)$;	$\mathrm{P}(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$	r/p	$r(1-p)/p^2$
index r, parameter p	$k = r, r+1, r+2, ...$		
(d) Poisson; $\mathcal{P}o(\mu)$	$\mathrm{P}(X = k) = \exp(-\mu)\mu^k/k!, \quad k = 0, 1, 2, ...$	μ	μ
(e) Uniform on (a, b); $U(a, b)$;	$f_X(x) = 1/(b-a), \quad a < x < b$	$(b+a)/2$	$(b-a)^2/12$
(f) Normal, $N(\mu, \sigma^2)$	$f_X(x) = (1/\sqrt{2\pi\sigma^2}) \exp(-(x-\mu)^2/2\sigma^2)$	μ	σ^2
(g) Exponential, $\mathcal{E}(\lambda)$	$f_X(x) = \lambda \exp(-\lambda x)$	$1/\lambda$	$1/\lambda^2$
rate parameter λ	$0 \le x < \infty$		
(g) Gamma, $G(\alpha, \lambda)$	$f_X(x) = \lambda^\alpha x^{\alpha-1} \exp(-\lambda x)/\Gamma(\alpha)$	α/λ	α/λ^2
shape α, rate λ	$0 \le x < \infty$		