1. *12 points total: 2 each sub-part*

(a) (i) Poisson mean 3; $P(X=3) \;=\; \exp(-3)3^3/3! \;=\; 0.224$

(a) (ii) Poisson mean $3 \times 2 = 6$; $P(X=6) \;=\; \exp(-6)6^6/6! \;=\; 0.161$

(a) (iii) F-kids in 2 weeks is Poisson mean 2; $P(X=0) \;=\; \exp(-2) \;=\; 0.135$

Or, can do it by waiting time, $T \;\sim\; \mathcal{E}(1)$, $P(T>2) = \exp(-2)$.

(b) (i) $P(3 \text{ each week} \mid 6 \text{ in 2 weeks}) \;=\; ((a)(i))^2/((a)(ii)) \;=\; (.224)^2/0.161 = 0.312 \;=\; 5/16$

(b) (ii) E-kids is Binomial(6,2/3): probability is $\binom{6}{3}(2/3)^3(1/3)^3 \;=\; 0.219 \;=\; 160/729$

(b) (iii) Probability 1 F-kid in first 4, then F-kid: $\binom{4}{1}(2/3)^3(1/3)^2 \;=\; 0.132 \;=\; 32/243$.

2. *10 points total*

(a) (2 points)

(i) $X \;\sim\; N(3,9)$. $\mathrm{E}(2X-4) \;=\; 6-4=2$. $\mathrm{var}(2X-4) \;=\; 4\mathrm{var}(X) = 36$

$Y \;\sim N(4,4)$. $\mathrm{E}(14-3Y) \;=\; 14-12 \;=\; 2$. $\mathrm{var}(14-3Y) \;=\; 9\mathrm{var}(Y) \;=\; 36$.

(ii) *(3 points)* Since distributions are Normal, need only same mean and variance.

$\mathrm{E}(aX+b) \;=\; 3a+b \;=\; 4 \;=\; \mathrm{E}(Y)$. $\quad \mathrm{var}(aX+b) \;=\; 9a^2 \;=\; 4 \;=\; \mathrm{var}(Y)$.

So $a = \pm 2/3$, $b \;=\; 4 \pm 2 \;=\; 6$ or 2.

(b) $X \;\sim\; G(1,1/3)$, $\mathrm{E}(X) \;=\; 3$, $\mathrm{var}(X) \;=\; 9$. $\quad Y \;\sim\; G(4,1)$. $\mathrm{E}(Y) \;=\; \mathrm{var}(Y) \;=\; 4$.

(i) *(2 points)*

Hence using part (a) $\mathrm{E}(2X-4) \;=\; 2$. $\mathrm{var}(2X-4) \;=\; 36$, $\quad \mathrm{E}(14-3Y) \;=\; 2$. $\mathrm{var}(14-3Y) \;=\; 36$.

(ii) *(3 points)*

$aX \;\sim\; G(1,1/3a)$, $bY \;\sim\; G(4,1/b)$ so we need $b=3a$.

Then $a(X+3Y) \;\sim\; G(5,1/3a) \;=\; 3aG(5,1)$, for any $a>0$

3. *10 points total.*

The jointly continuous random variables X and Y have joint density function

$$f_{X,Y}(x,y) = 6y^2/x^2 \text{ on } 0<y<x<1, \text{ and } f_{X,Y}(x,y)=0 \text{ for all other } (x,y).$$

(a) *(4 points)*

$$f_X(x) \;=\; \int_{y=0}^{x} 6y^2/x^2 dy \;=\; (6/x^2)[y^3/3]_0^x \;=\; 2x \quad \text{on } 0<x<1$$

(b) *(2 points)* X and Y cannot be independent since $Y<X$.

(Also, $f_X(x) \;=\; 2x$ is not a factor of $f_{X,Y}(x,y)$.)

(c) *(4 points)* It is given that the marginal density of Y is $f_Y(y) = 6y(1-y)$ on $0<y<1$.

$$\mathrm{E}(1/(1-Y)) \;=\; \int_{y=0}^{1}(1-y)^{-1}6y(1-y)\,dy \;=\; \int_0^1 6y\,dy \;=\; [3y^2]_0^1 \;=\; 3$$