Explain/justify all your your answers.

1. *12 points total: 2 each sub-part*

Over 4 weeks in August, a small bilingual pre-school in Ottawa (Canada) accepts children in the order their parents enroll them. Children from English-speaking families (E-kids) enroll as a Poisson process rate 2 per week, and children from French-speaking families (F-kids) enroll as a Poisson process rate 1 per week. Hence enrollments in total occur as a Poisson process rate 3 per week, and each child to be enrolled is French-speaking (F-kid) with probability 1/3, and is English-speaking (E-kid) with probability 2/3, independently of the type of other enrollees. (**You do not need to show this**.)

(a) (i) What is the probability that a total of three kids enroll in the first week of August ?

(ii) What is the probability that a total of six kids enroll in the last two weeks of August ?

(iii) What is the probability that the waiting time until the enrollment of the first F-kid is more than 2 weeks?

(b) Given that a total of 6 kids enrolled over the first two weeks of August:

(i) What is the probability that 3 kids enrolled in each of these two weeks?

(ii) What is the probability that 3 of these 6 were E-kids and 3 were F-kids?

(iii) What is the probability that, of these 6 kids, the 5 th was the second F-kid to enroll?

(**Note:** these are three *separate* questions, each given a total of 6 kids in these two weeks.).

2. *10 points total*

(a) The random variable X has a Normal distribution with mean 3 and variance 9: $X \sim N(3,9)$.
The random variable Y has a Normal distribution with mean 4 and variance 4: $Y \sim N(4,4)$.

(i) *(2 points)* What are the mean and variance of $2X-4$?
What are the mean and variance of $14-3Y$?

(ii) *(3 points)* For what values of a and b does $aX+b$ have the same distribution as Y?

(b) The random variable X has a Gamma distribution with shape parameter 1 and rate 1/3: $X \sim G(1,1/3)$.
The random variable Y has a Gamma distribution with shape parameter 4 and rate 1: $Y \sim G(4,1)$.
X and Y are independent, and you may assume that the sum of independent Gamma random variables is a Gamma distributed random variable if and only if they have the same rate parameter.

(i) *(2 points)* What are the mean and variance of $2X-4$?
What are the mean and variance of $14-3Y$?
(Hint: first figure the mean and variance of X and Y, and see (a).)

(ii) *(3 points)* For what values of a and b does $aX+bY$ have a Gamma distribution, and what are the parameters of this Gamma distribution?

3. *10 points total.*

The jointly continuous random variables X and Y have joint density function

$$f_{X,Y}(x,y) = 6y^2/x^2 \text{ on } 0<y<x<1, \text{ and } f_{X,Y}(x,y)=0 \text{ for all other } (x,y).$$

(a) *(4 points)* Show that the marginal density of X is $f_X(x)=2x$, on $0<x<1$.

(b) *(2 points)* Are X and Y independent? Why?/Why not?

(c) *(4 points)* It can be shown that the marginal density of Y is $f_Y(y)=6y(1-y)$ on $0<y<1$. (**You do not need to show this**.) Find the expected value of $1/(1-Y)$?