

Lecture 9: Discrete random variables Ross 4.1,4.2. (There are no notes for lectures 7,8: 10/8, 10/10)

9.1 Definitions and examples

- (i) **Definition:** A random variable X is a real-valued function on the sample space.
- (ii) Example: the number of heads in 10 tosses of a fair coin.
- (iii) Example: the number of children of blood-type A out of n , from marriages of $AB \times O$ parents.
- (iv) Example: the number of red pea plants out of n , from crosses of $RW \times RW$ pea plants.
- (v) Example: the number of red fish, in sampling k fish without replacement, from a pond in which there are N fish of which n are red.
- (vi) Example: the number of traffic accidents in a large city in a year.

9.2: Discrete Random variables

- (i) **Definition:** A random variable X is *discrete* if it can take only a discrete set of values.
- (ii) Example (ii) above: X takes a value in $\mathcal{X} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
- (iii) Example (iii) above: X takes a value in $\mathcal{X} = \{0, 1, 2, \dots, n\}$.
- (iv) Example (iv) above: ditto.
- (v) Example (v) above: X takes values in $\mathcal{X} = \{\max(0, k + n - N), \dots, \max(k, n)\}$.
- (vi) Example (vi) above: X takes values in $\mathcal{X} = \{0, 1, 2, \dots\} = \{0\} \cup \mathbb{Z}_+$.

9.3 Probability mass function

- (i) **Definition:** The *probability mass function* (p.m.f.) of a discrete random variable X is the set of probabilities $P(X = x)$ for each of the values $x \in \mathcal{X}$ that X can take.
- (ii) Example (ii): $P(X = x) = \binom{10}{x}(1/2)^{10}$ for $x = 0, 1, 2, \dots, 10$.
- (iii) Example (iii): $P(X = x) = \binom{n}{x}(1/2)^n$ for $x = 0, 1, 2, \dots, n$.
- (iv) Example (iv): $P(X = x) = \binom{n}{x}(3/4)^x(1/4)^{n-x}$ for $x = 0, 1, 2, \dots, n$.
- (v) Example (v): $P(X = x) = \binom{n}{x} \binom{N-n}{k-x} / \binom{N}{k}$ for $x = \max(0, k + n - N), \dots, \max(k, n)$.
- (vi) Example (vi): $P(X = x) = \exp(-\lambda)\lambda^x/x!$ for $x=0,1,2,3,4,\dots$

9.4 Properties of probability mass functions

- (i) $P(X = x) \geq 0$ for each $x \in \mathcal{X}$

In fact, for discrete random variables, we can require $P(X = x) > 0$ for each $x \in \mathcal{X}$.

- (ii) $\sum_x P(X = x) = 1$ where the sum is over all $x \in \mathcal{X}$.

9.5 Names of some standard probability mass functions

- (i) *Binomial*: Examples (ii), (iii), and (iv) are Binomial random variables, with *parameter* $(1/2)$, $(1/2)$ and $(3/4)$ respectively.
- (ii) *Bernoulli*: If $n = 1$, examples (iii) and (iv) are Bernoulli random variables.
- (iii) *Multinomial*: If there are more than two types (for example, number of each of types A , B , AB and O in sample size n from a population) then we have a Multinomial random variable.
- (iv) *Hypergeometric*: In example (v), X is a Hypergeometric random variable.
- (v) *Poisson*: In example (vi), X is a Poisson random variable, with parameter λ .

Lecture 10: Continuous random variables Ross 5.1

10.1 Definitions and examples

- (i) **Definition:** A *continuous* random variable X is one that takes values in $(-\infty, \infty)$. That is, in principle: some values may be impossible.
- (ii) Example: A random number between a and b : values in the interval (a, b) .
- (iii) Example: The waiting time until the bus arrives: values in $(0, \infty)$.
- (iv) Example: The height of an individual, relative to the population average: values in $(-\infty, +\infty)$ (in principle).

10.2 The probability density function: definition and basic properties.

- (i) **Definition:** The *probability density function* (p.d.f.) of a continuous random variable X is a non-negative function f defined for all values x in $(-\infty, \infty)$ such that for any subset B for which $X \in B$ is an event

$$P(X \in B) = \int_B f(x) dx$$

- (ii) In fact, events can be made up of unions and intersections of intervals of the form $(a, b]$:

$$P(a < X \leq b) = \int_a^b f(x) dx$$

- (iii) Note the value at the boundary does not matter:

$$P(X = a) = \int_a^a f(x) dx = 0$$

for any continuous random variable.

- (iv) Note: $f(x) = 0$ is possible for some x -values (see the p.m.f).
For example, if $X \geq 0$ (as in the waiting-time example), $f(x) = 0$, if $x < 0$.
- (v) X takes some value in $(-\infty, \infty)$ so

$$1 = P(-\infty < X < \infty) = \int_{-\infty}^{\infty} f(x) dx$$

10.3 Examples: of the probability density function

- (i) In general: any non-negative real-valued function f on the real line such that $\int_{-\infty}^{\infty} f(x) dx = 1$.
- (ii) Example (ii) above: *Uniform p.d.f.*:

$$f(x) = \frac{1}{(b-a)} \text{ for } a \leq x \leq b \text{ and } f(x) = 0 \text{ otherwise.}$$

- (iii) Example (iii) above: *Exponential p.d.f.* (with rate parameter λ):

$$f(x) = \lambda \exp(-\lambda x) \text{ for } x \geq 0 \text{ and } f(x) = 0 \text{ if } x < 0.$$

- (iv) Example (iv) above: *Normal p.d.f.* (with parameters μ and σ^2):

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \text{ for } -\infty < x < \infty.$$