MATH/STAT 394 A: FALL 2008: Information sheet for Midterm-2: 8:30-9:20, Wed Nov 12

1. Permutations and combinations

There are $n! = \prod_{i=1}^{n} i = 1.2.3.4.\ldots.n$ permutations of n objects.

There are $\binom{n}{k} = n!/(k!(n-k)!)$ ways of choosing a given k objects from n.

2. Joint and conditional probabilities

If C and D are any events: $P(C \bigcup D) = P(C) + P(D) - P(C \bigcap D)$.

The conditional probability of C given D is $P(C \mid D) = P(C \bigcap D) / P(D)$.

C and D are independent if $P(C \bigcap D) = P(C).P(D)$.

3. Laws and theorems

Suppose $E_1, \ldots, E_k$ is a partition of Ω. That is $E_i \bigcap E_j$ is empty, and $E_1 \bigcup E_2 \bigcup \ldots \bigcup E_k = \Omega$.

The law of total probability states that: $P(D) = \sum_{j=1}^{k} P(D \bigcap E_j) = \sum_{j=1}^{k} P(D \mid E_j)\, P(E_j)$

Bayes' Theorem states that: $P(E_i \mid D) = P(D \mid E_i)\, P(E_i)/P(D)$

4. Random variables and distributions	**discrete (mass)**	**continuous (density)**
Probability mass/density function	pmf: $P(X = x) = p_X(x)$	pdf: $f_X(x)$
defined for	all x with $p_X(x) > 0$	$-\infty < x < \infty$
Expectation E(X)	$\sum_x x p_X(x)$	$\int_{-\infty}^{\infty} x f_X(x) dx$
Result: E($g(X)$)	$\sum_x g(x) p_X(x)$	$\int_{-\infty}^{\infty} g(x) f_X(x) dx$

Variance: $\mathrm{E}((X - (\mathrm{E}(X)))^2) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

5. Standard distributions: p_X or f_X	values	mean E(X)	variance
(a) Binomial; $B(n,p)$; index n, parameter p;			
$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$	$k = 0, 1, 2, ..., n$	np	$np(1-p)$
(b) Geometric; $G(p)$; parameter p;			
$P(X = k) = p(1-p)^{k-1}$	$k = 1, 2, 3, 4, ...$	$1/p$	$(1-p)/p^2$
(c) Negative Binomial; $NegB(r,p)$			
$P(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$	$k = r, r+1, r+2, ...$	r/p	$r(1-p)/p^2$
(d) Poisson; $Po(\mu)$; parameter μ;			
$P(X = k) = \exp(-\mu)\mu^k/k!$	$k = 0, 1, 2, 3, ...$	μ	μ
(e) Hypergeometric (N, m, n);			
$P(X = k) = \binom{m}{k}\binom{N-m}{n-k}/\binom{N}{n}$	$k = 0, ..., n$ $k \geq m + n - N,\ k \leq m$	nm/N	—-
(f) Uniform on interval (a,b); $U(a,b)$;			
$f_X(x) = 1/(b-a)$	$a < x < b$	$(a+b)/2$	$(b-a)^2/12$
(g) Exponential, $E(\lambda)$, rate parameter λ;			
$f_X(x) = \lambda \exp(-\lambda x)$	$0 \leq x < \infty$	$1/\lambda$	$1/\lambda^2$

6. Summation of series:

(a) $\sum_{i=0}^{n-1} x^i = (1 + x + x^2 + ... + x^{n-1}) = (1 - x^n)/(1 - x)$

(b) $\sum_{i=0}^{\infty} x^i/i! = (1 + x + x^2/2 + x^3/6 + ...) = \exp(x)$