Stat581: Fall 2004: Week 1 Quiz: solns

1 [4 pts]. $E(U^c) = \int_0^1 u^c du$ **if the integral converges. The indefinite integral of** u^c **is** $u^{c+1}/(c+1)$ **if** $c \neq -1$ **and** $\log(u)$ **if** $c = -1$**. So** $E(U^c)$ **is finite iff** $c > -1$ **and then is** $1/(c+1)$.

2 [3 pts].
2(a) Let F_n **be cdf of** V_n **and** F **be cdf of** V**, then** $F_n(v) \to F(v)$ **at all** $v \in \Re$ **at which** F **is continuous.**

2(b) For all $\epsilon > 0$, $\Pr(|V_n - V| > \epsilon) \longrightarrow 0$ **as** $n \to \infty$.

3 [4 pts] For small ϵ, $\int_0^\epsilon (x(1-x)^{-1+1/n}dx \geq (1/2)\int_0^\epsilon x^{-1+1/n}dx = n\epsilon^{1/n}/2 \to \infty$ **as** $n \to \infty$**, so the normalizing const of this density** $\to 0$**. Hence** $\Pr(\epsilon < X_n < 1-\epsilon) \to 0$**. By symmetry** $\Pr(X_n < \epsilon) = \Pr(X_n > 1-\epsilon)$, $\forall\ n$**. So** $F_n(x) \to 1/2 = F(x)$ **for** $0 < x < 1$**, as required.**

4 [6 pts]. Note here we are given the joint dsn, since all are defined in terms of the single r.v. U.

4(a) $\Pr(Y_n = 0) = 1 - 1/n$**, so** Y_n **converges in probability to 0.**

4(b) Hence it also converges in dsn to 0.

4(c) $e_n = 1$ **for all** n.
So $\lim(E(Y_n)) = 1 > 0 = E(\lim(Y_n))$ **—an example of strict inequality in Fatou's lemma.**

5 [2 pts]. If X_i, $i = 1, 2, ...n$ **are i.i.d with** $E(X_i) = \mu$ **and** $\text{var}(X_i) = \sigma^2 < \infty$**, and** $\overline{X_n} = n^{-1}\sum_{i=1}^n X_i$**, then** $\sqrt{n}(\overline{X_n} - \mu)$ **converges in distribution to** $N(0, \sigma^2)$.

6 [6 pts]
6(a) $T_n \sim Bin(n, p)$.

6(b) W_n **converges in probability to** p **(Weak Law of Large Numbers).**

6(c) $\sqrt{n}(W_n - p)$ **converges in dsn to** $N(0, p(1-p))$**, by CLT, see #5.**

6(d) Direct computation of Fisher information gives lower bound as $p(1-p)/n$ **– hence** W_n **is MVUE of** p