Day 3 Quiz: Oct 4, 2004. STATISTICS 581

1. Suppose that $U \sim U(0,1)$. For what values of $c \in \Re$ is it true that $\mathrm{E}(U^c) < \infty$? For the values of c for which the integral is finite, compute it explicitly.

2. For random variables $V_n, n = 1, 2, ...,$ and V, define what is meant by
(a) V_n converges in distribution to V
(b) V_n converges in probability to V.

3. Suppose that X_n has the symmetric beta distribution $Be(1/n, 1/n)$ with density on $0 \le x \le 1$ proportional to $(x(1-x))^{-(n-1)/n}$. Show that X_n converges in distribution to the discrete random variable X with $\Pr(X = 0) = \Pr(X = 1) = 1/2$.
(Hint: Consider $\Pr(\epsilon < X_n < 1 - \epsilon)$ for any $\epsilon > 0$, as $n \to \infty$.

4. Suppose that U is a random variable with a $U(0,1)$ distribution.
For each n, $n = 1, 2, ...,$ define $Y_n = n$, if $0 < U < 1/n$, and $Y_n = 0$ otherwise.
(a) Does Y_n converge in probability? If so, identify the limit random variable Y.
(b) Does Y_n converge in distribution? If so, identify the limit random variable Y.
(c) Compute $e_n = \mathrm{E}(Y_n)$. Does the sequence e_n converge?
Does it converge to the expectation of the random variable Y in (a) or (b)?

5. State any form of the Central Limit Theorem.

6. Suppose that $X_1, ..., X_n$ are independent Bernoulli random variables, each satisfying $\Pr(X_i = 1) = p$ and $\Pr(X_i = 0) = 1 - p$.
(a) Let $T_n = X_1 + ... + X_n$. What is the distribution of T_n?
(b) Let $W_n = T_n/n$. Does W_n converge in probability? If so, to what?
(c) Does $\sqrt{n}(W_n - p)$ converge in distribution? If so, to what?
(d) What is the Cramér lower bound on the variance of unbiased estimators of p ?