Stat 581 Homework 1: Due October 6, 2004

1. Let X and Y be independent random variables, each with a uniform $U(0,1)$ distribution. Let $V = X - Y$ and $W = \max(X, Y)$.

(a) What is the range (support) of the distribution of (V, W)?

(b) Find the joint density function $f_{V,W}(v, w)$ of (V, W).

(c) Are V and W independent?

2. (a) Suppose that V_n has the discrete uniform distribution

$$\Pr(V_n = j/n) \quad = \quad 1/n, \text{ for } j = 1, 2, 3, ..., n$$

Does the sequence of random variables $V_n, n = 1, 2, ...$ converge in distribution? Does it converge in probability?

(b) Suppose that the sequence of real-valued random variables X_n with distribution function $F_n(\cdot)$ converges in distribution to X, where the distribution function $F(\cdot)$ of X is continuous. Show that F_n converges uniformly to F on the real line.

3. Suppose that for a real-valued parameter θ and for $0 \le v \le 1$, $0 \le w \le 1$,

$$f_\theta(v, w) \quad = \quad (1 + \theta(1 - 2v)(1 - 2w))$$

and $f_\theta(v, w) = 0$ for all other (v, w).

(a) For what values of θ is f_θ a density for a pair of random variables (V, W)?

(b) For this set of θ, find the corresponding distribution function F_θ, and show that V and W each has a uniform $U(0,1)$ distribution.

(c) For (V, W) with distribution function F_θ, find the correlation ρ between V and W. Does this show any difficuly with this family of distributions as a model of dependence?

4. Let Y_n, $n = 1, 2, \ldots$ be independent and identically distributed uniform r.vs on $0 \le y \le \theta$: $Y_n \sim U(0, \theta)$.

(a) Let $V_n = \sqrt{n}(n^{-1} \sum_1^n Y_n - \theta/2)$.
Does V_n converge in distribution? If so, to what?

(b) Let $W_n = n \min_{1 \le j \le n} Y_j$.
Does W_n converge in distribution? If so, to what?