Stat 581 Homework 3: Due October 22, 2003

5. Let F be a cdf, and for $0 < t < 1$ define $F^{-1}(t) = \inf\{x : F(x) \geq t\}$. Show that

(a) F^{-1} is left-continuous

(b) $F^{-1}(F(x)) \leq x$, $-\infty < x < \infty$

(c) $F(F^{-1}(t)) \geq t$, $0 < t < 1$

(d) $F(x) \geq t$ if and only if $x \geq F^{-1}(t)$.

6. For distribution functions F_n $(n = 1, 2, 3...)$ and F, let $F_n(x) \to F(x)$, $-\infty < x < \infty$. Suppose t_0 is s.t. $F_n^{-1}(t_0) \not\to F^{-1}(t_0)$. Choose ϵ s.t. $|F_n^{-1}(t_0) - F^{-1}(t_0)| > \epsilon$ for infinitely many n.

(a) Show that $F_n^{-1}(t_0) > F^{-1}(t_0) + \epsilon$ for infinitely many n
(Hint: show $F_n^{-1}(t_0) < F^{-1}(t_0) - \epsilon$ i.o. is impossible.)

(b) Deduce that $t_0 = F(F^{-1}(t_0)$ and that F is therefore flat in a right-neighborhood of $F^{-1}(t_0)$.

(c) Show that there are at most countably many points in the set

$$\{t : \ 0 < t < 1, \ F_n^{-1}(t) \not\to F^{-1}(t), n \to \infty\}$$

Hint: a non-decreasing function has at most countably many discontinuities; why?