5.2.4. Suppose a random sample of size n is drawn from the probability model

$$p_X(k;\theta) = \frac{\theta^{2k}e^{-\theta^2}}{k!}, \quad k = 0, 1, 2, \ldots$$

Find a formula for the maximum likelihood estimator, $\hat{\theta}$.

5.2.6. Use the method of maximum likelihood to estimate θ in the pdf

$$f_Y(y;\theta) = \frac{\theta}{2\sqrt{y}}e^{-\theta\sqrt{y}}, \quad y > 0$$

Evaluate θ_e for the following random sample of size 4: $Y_1 = 6.2$, $Y_2 = 7.0$, $Y_3 = 2.5$, and $Y_4 = 4.2$.

5.2.9. **(a)** Based on the random sample $Y_1 = 6.3$, $Y_2 = 1.8$, $Y_3 = 14.2$, and $Y_4 = 7.6$, use the method of maximum likelihood to estimate the parameter θ in the uniform pdf

$$f_Y(y;\theta) = \frac{1}{\theta}, \quad 0 \le y \le \theta$$

(b) Suppose the random sample in Part (a) represents the two-parameter uniform pdf

$$f_Y(y;\theta_1,\theta_2) = \frac{1}{\theta_2 - \theta_1}, \quad \theta_1 \le y \le \theta_2$$

Find the maximum likelihood estimates for θ_1 and θ_2.

5.2.11. A random sample of size n is taken from the pdf

$$f_Y(y;\theta) = 2y\theta^2, \quad 0 \le y \le \frac{1}{\theta}$$

Find an expression for $\hat{\theta}$, the maximum likelihood estimator for θ.

5.2.12. If the random variable Y denotes an individual's income, Pareto's law claims that $P(Y \ge y) = \left(\frac{k}{y}\right)^{\theta}$, where k is the entire population's minimum income. It follows that $F_Y(y) = 1 - \left(\frac{k}{y}\right)^{\theta}$, and, by differentiation,

$$f_Y(y;\theta) = \theta k^{\theta}\left(\frac{1}{y}\right)^{\theta+1}, \quad y \ge k; \quad \theta \ge 1$$

Assume k is known. Find the maximum likelihood estimator for θ if income information has been collected on a random sample of 25 individuals.

5.2.14. Suppose a random sample of size n is drawn from a normal pdf where the mean μ is known but the variance σ^2 is unknown. Use the method of maximum likelihood to find a formula for $\hat{\sigma}^2$. Compare your answer to the maximum likelihood estimator found in Example 5.2.4.