QUESTIONS

5.8.1. Suppose that X is a geometric random variable, where $p_X(k|\theta) = (1 - \theta)^k \theta$, $k = 1, 2, \ldots$. Assume that the prior distribution for θ is the beta pdf with parameters r and s. Find the posterior distribution for θ.

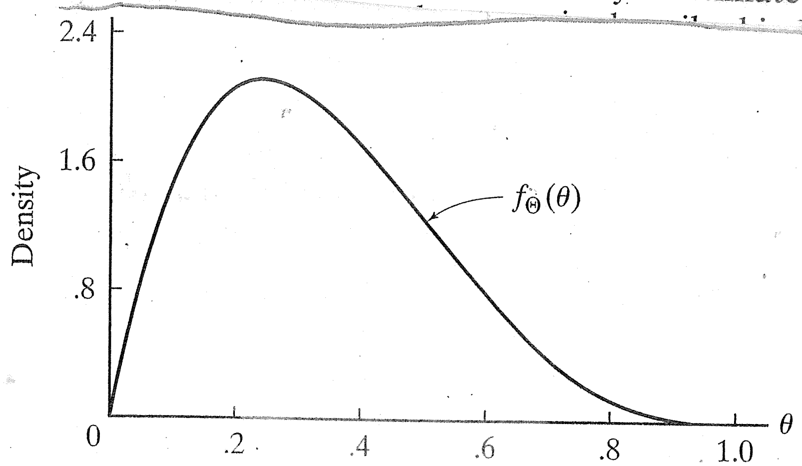
5.8.4. What is the squared-error loss Bayes estimate for the parameter θ in a binomial pdf, where θ has a uniform distribution—that is, a noninformative prior? (Recall that a uniform prior is a beta pdf for which $r = s = 1$).

5.8.5. In Questions 5.8.2–5.8.4, is the Bayes estimate unbiased? Is it asymptotically unbiased?

5.8.6. Suppose that Y is a gamma random variable with parameters r and θ and the prior is also gamma with parameters s and μ. Show that the posterior pdf is gamma with parameters $r + s$ and $y + \mu$.

5.8.7. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a gamma pdf with parameters r and θ, where the prior distribution assigned to θ is the gamma pdf with parameters s and μ. Let $W = Y_1 + Y_2 + \cdots + Y_n$. Find the posterior pdf for θ.

5.8.8. Find the squared-error loss Bayes estimate for θ in Question 5.8.7.



Beta Density

One such probability model whose shape would comply with the restraints that Max is imposing is the *beta pdf*. Written with Θ as the random variable, the (two-parameter) beta pdf is given by

$$f_\Theta(\theta) = \frac{\Gamma(r + s)}{\Gamma(r)\Gamma(s)} \theta^{r-1}(1 - \theta)^{s-1}, \quad 0 \le \theta \le 1$$

The beta distribution with $r = 2$ and $s = 4$ is pictured in Figure 5.8.1. By choosing different values for r and s, $f_\Theta(\theta)$ can be skewed more sharply to the right or to the left, and the bulk of the distribution can be concentrated close to zero or close to one.