QUESTIONS

5.6.1. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the geometric distribution, $p_X(k; p) = (1 - p)^{k-1}p, k = 1, 2, \ldots$. Show that $\hat{p} = \sum_{i=1}^{n} X_i$ is sufficient for p.

5.6.2. Suppose a random sample of size n is drawn from the pdf,

$$f_Y(y; \theta) = e^{-(y-\theta)}, \quad \theta \leq y$$

(a) Show that $\hat{\theta} = Y_{\min}$ is sufficient for the threshold parameter θ.
(b) Show that $Y_{\max}$ is not sufficient for θ.

5.6.5. Show that $\hat{\sigma}^2 = \sum_{i=1}^{n} Y_i^2$ is sufficient for σ^2 if $Y_1, Y_2, \ldots, Y_n$ is a random sample from a normal pdf with $\mu = 0$.

5.5.2. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from $f_Y(y; \theta) = \frac{1}{\theta}e^{-y/\theta}, y > 0$. Compare the Cramér-Rao lower bound for $f_Y(y; \theta)$ to the variance of the maximum likelihood estimator for θ, $\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. Is $\overline{Y}$ a best estimator for θ?

5.5.3. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the Poisson distribution, $p_X(k; \lambda) = \frac{e^{-\lambda}\lambda^k}{k!}, k = 0, 1, \ldots$. Show that $\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is an efficient estimator for λ.

5.5.4. Suppose a random sample of size n is taken from a normal distribution with mean μ and variance σ^2, where σ^2 is known. Compare the Cramér-Rao lower bound for $f_Y(y; \mu)$ with the variance of $\hat{\mu} = \overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. Is $\overline{Y}$ an efficient estimator for μ?