STAT 341 - Elizabeth Thompson

Homework 5 Solutions

LM: 3.7.11, 3.7.22, 3.7.44

(3.7.11)

$$\begin{aligned} P(Y < 3X) &= \int_0^\infty \int_x^{3x} 2e^{-(x+y)} dydx \\ &= 2\int_0^\infty e^{-x} \int_x^{3x} e^{-y} dydx \\ &= 2\int_0^\infty e^{-x} \left(\left[-e^{-y} \right]_x^{3x} \right) dx \\ &= 2\int_0^\infty e^{-x} \left(\left[-e^{-3x} + e^{-x} \right] \right) dx \\ &= 2\int_0^\infty \left(-e^{-4x} + e^{-2} \right) dx \\ &= 2\left[\frac{1}{4}e^{-4x} - \frac{1}{2}e^{-2x} \right]_0^\infty = \frac{1}{2} \end{aligned}$$

(3.7.22)

Note that in the shaded region, $0 < x < y$.

$$f_Y(y) = \int_0^y 2e^{-x}e^{-y}dx = -2e^{-x}e^{-y}|_0^y = 2e^{-y} - 2e^{-2y}, \quad 0 \leq y$$

(3.7.44)

$$F_X(x) = \int_0^x \frac{t}{2}dt = \frac{x^2}{4}, \quad F_Y(y) = \int_0^y 2tdt = y^2$$

By independence,

$$F_{X,Y}(x,y) = F_X(x)F_Y(y) = \frac{x^2y^2}{4}, \quad 0 \leq x \leq 2, 0 \leq y \leq 1$$