STAT 341 - Elizabeth Thompson

Homework 4 Solutions

4.2.10, 4.2.26, 4.3.10, 3.12.5 (b,c,d), 3.12.14, 4.6.1

(4.2.10)

The average number of fatalities per corps-year = $\frac{109(0)+65(1)+22(2)+3(3)+1(4)}{200} = 0.61$, so the presumed Poisson mode is $p_x(k) = \frac{e^{-0.61}(0.61)^k}{k!}, k = 0, 1, \ldots$.

No. Deaths (k)	Frequency	Proportion	$p_x(k)$
0	109	0.545	0.5434
1	65	0.325	0.3314
2	22	0.110	0.1011
3	3	0.015	0.0206
4+	1	0.005	0.0035
Total	200	1.000	1.000

(4.2.26)

(a)Yes, because the assumptions are probably satisfied - crashes are independent events and the crash rate is likely to remain constant.

(b) Since $\lambda = 2.5$ crashes per year, $P(X \geq 4) = 1 - P(X \leq 3) = 1 - \sum_{k=0}^{3} \frac{e^{-2.5}(2.5)^3}{k!} = 0.24$

(c) Let Y = interval (in years) between next two crashes. $P(Y < 0.25) = \int_0^{0.25} 2.5e^{-2.5y}dy = 1 - 0.535 = 0.465$

(4.3.10)

(a) Let X = number of shots made in next 100 attempts. $p = .70$

$$P(75 \leq X \leq 80) = \sum_{k=75}^{80} {}_{100}C_k(0.70)^k(0.30)^{100-k}$$

(b) $np = 100(0.70) = 70$ and $np(1-p) = 100(0.70)(0.30) = 21$. With continuity correction,

$$P(74.5 \leq X \leq 80.5) = P\left(\frac{74.5 - 70}{\sqrt{21}} \leq \frac{X - 70}{\sqrt{21}} \leq \frac{80.5 - 70}{\sqrt{21}}\right) = P(0.98 \leq Z \leq 2.29) = 0.1525$$

(3.12.5 (b,c,d))

(b) Exponential with $\lambda = 2$.

(c) Binomial with $n = 4$ and $p = 1/2$

(d) Geometric with $p = 0.3$

(3.12.14)

$$M_Y^{(1)} = \frac{d}{dt}(1-t/\lambda)^{-r} = (r/\lambda)(1-t/\lambda)^{-r-1}$$

$$M_Y^{(2)} = \frac{d}{dt}(-r/\lambda)(1-t/\lambda)^{-r-1} = \frac{r(r+1)}{\lambda^2}(1-t/\lambda)^{-r-2}$$

Continuing in this fashion yields,

$$M_Y^{(k)} = \frac{(r+k-1)!}{\lambda^k(r-1)!}(1-t/\lambda)^{-r-k}$$

$$E[Y^k] = M_Y^{(k)}(0) = \frac{(r+k-1)!}{\lambda^k(r-1)!}$$

(4.6.1)

Let Y_i = lifetime of the ith gauge, $i = 1, 2, 3$. $f_{Y_i}(y) = 0.001e^{-0.001y}, y > 0$. $Y = Y_1 + Y_2 + Y_3$ is the lifetime of the system. By Theorem 4.6.1,

$$f_Y(y) = \frac{(0.001)^3}{2}y^2e^{-0.001y}, y > 0$$