STAT 341 - Elizabeth Thompson

Homework 2 Solutions

3.5.4, 3.5.7, 3.6.9, 3.6.12, 5.4.9, 5.4.18

(3.5.4)

$$E[A] = -5 + 0 \times \frac{{}_2C_0 \times_4 C_2}{{}_6C_2} + 2 \times \frac{{}_2C_1 \times_4 C_1}{{}_6C_2} + 10 \times \frac{{}_2C_2 \times_4 C_0}{{}_6C_2} = -49/15$$

$$E[B] = -5 + 0 \times \frac{{}_2C_0 \times_4 C_2}{{}_6C_2} + 1 \times \frac{{}_2C_1 \times_4 C_1}{{}_6C_2} + 20 \times \frac{{}_2C_2 \times_4 C_0}{{}_6C_2} = -47/15$$

Rule B has the better expected payoff.

(3.5.7)

This is a hypergeometric problem where r = number of students needing vaccinations = 125, and w = number of students already vaccinated = 642 - 125 = 517. An absenteeism rate of 12% corresponds to a sample of $n = 0.12 \times 642 = 77$ missing students. The expected number of unvaccinated students who are absent when the physician visits is $\frac{125 \times 77}{125+517} = 15$

(3.6.9)

Let Y be Frankie's selection. Johnny wants to choose k so that $E[(Y-k)^2]$ is minimized. Let $\mu = E[Y]$.

$$\begin{aligned} E[(Y-k)^2] &= E[((Y-\mu)+(\mu-k))^2] \\ &= E[(Y-\mu)^2] + (\mu-k)^2 + 2(\mu-k)E[(Y-\mu)] \\ &= V[Y] + (\mu-k)^2 \end{aligned}$$

Since $E[(Y-\mu)] = 0$, the minimum occurs when $k = \mu = (a+b)/2$.

(3.6.12)

$E[Y] = 1/2$ and $V[Y] = 1/4$.

$$P(Y > E[Y] + 2\sqrt{V[Y]}) = P\left(Y > \frac{1}{2} + 2\sqrt{\frac{1}{4}}\right) = P\left(Y > \frac{3}{2}\right)$$

$$= \int_{3/2}^{\infty} 2e^{-2y} dy = e^{-3} = 0.0498$$

(5.4.9)

$$E[Y] = 2\int_0^{1/\theta} y^2\theta^2 dy = \frac{2}{3}\left(\frac{1}{\theta}\right)$$

$$E[c(Y_1 + 2Y_2)] = c[E(Y_1) + 2E(Y_2)] = c\left[\frac{2}{3}\left(\frac{1}{\theta}\right) + \frac{4}{3}\left(\frac{1}{\theta}\right)\right] = 2c\left(\frac{1}{\theta}\right)$$

For the estimator to be unbiased, $c = 1/2$.

(5.4.18)

$V\left(\frac{6}{5} \cdot Y_{max}\right) = \frac{\theta^2}{35}$ See Example 5.4.7

$$V(Y_{max}) = V\left[\frac{5}{6}\left(\frac{6}{5}Y_{max}\right)\right] = \frac{25}{36}V\left(\frac{6}{5} \cdot Y_{max}\right) = 5\frac{\theta^2}{252}$$

By symmetry, $V(Y_{min}) = V(Y_{max})$.

$$V(6 \cdot Y_{min}) = 36V(Y_{min}) = \frac{5\theta^2}{7}$$

Thus $\frac{6}{5} \cdot Y_{max}$ has lower variance. This makes sense intuitively since, $V(Y_{min}) = V(Y_{max})$. Thus efficiency depends on the size of the constant to make the estimator unbiased.