STAT 341 WINTER 2010 **Midterm 2: 11:30-12:20 Monday, February 1, 2010**

Explain/justify ALL your answers.

1. (*12 points: 4 each part*)

X_1, X_2, and X_3 are a sample of three independent realizations from a distribution with mean θ and variance 1. Consider two estimators of θ; $T_1 = (X_1 + X_2 + X_3)/3$ and $T_2 = (4X_1 + 3X_2 + X_3)/9$.

(a) Which estimator (if either) is an unbiased estimator of θ?

(b) Which estimator (if either) has smaller variance?

(c) Which estimator (if either) has smaller MSE?

2. (*12 points: 3 each part*)

A company maintains a very large number of machines, and the number that break down each month can be modeled as the outcome of a Poisson random variable with mean θ, and successive months are independent. The monthly cost to the maintenance company is proportional to the *square* of the number of breakdowns in that month (due to overtime, delays in spare parts, etc.).

(a) If X is distributed as a Poisson random variable with mean θ, show that $\mathrm{E}(X^2) = \theta^2 + \theta$.

(b) If X is distributed as a Poisson random variable with mean θ, **it can be shown that** $\mathrm{E}(X^3) = \theta(\theta^2+3\theta+1)$ and $\mathrm{E}(X^4) = \theta(\theta^3 + 6\theta^2 + 7\theta + 1)$. (**Do NOT try to show this.**). **Assuming these formulas**, find an expression for $\mathrm{var}(X^2)$ in terms of θ. (Do **not** try to tidy it up– it is just messy.)

(c) The company manager decides to observe the numbers of breakdowns for n months; that is the outcomes of an n-sample $X_1, ..., X_n$, from a Poisson $\mathcal{P}\mathrm{o}(\theta)$ distribution. He will then estimate $\theta^2 + \theta$ using the statistic $T = (1/n).\sum_{i=1}^{n} X_i^2$. Show that T is an unbiased estimator of $\theta^2 + \theta$.

(d) Using Chebychev's inequality, show $T = (1/n).\sum_{i=1}^{n} X_i^2$ is a consistent estimator of $\theta^2 + \theta$

3. (*12 points: 4 each part*)

Continuing the story of the previous question the manager consults a statistician who tells him he should use the Method of Moments (MoM) to find his estimator of $\theta^2 + \theta$.

(a) If $X_1, ..., X_n$ are i.i.d. Poisson with mean θ,
show the MoM estimator of $\theta^2 + \theta$ is $W = \overline{X_n}^2 + \overline{X_n}$, where $\overline{X_n} = (1/n)\sum_{i=1}^{n} X_i$.

(b) If $X_1, ..., X_n$ are i.i.d. Poisson with mean θ, and $\overline{X_n} = (1/n)\sum_{i=1}^{n} X_i$, show $\mathrm{E}(\overline{X_n}^2) = \theta^2 + \theta/n$

(c) Show the MoM estimator W is a biased estimator of $\theta^2 + \theta$, but asymptotically unbiased.