**Homework 8; Due 11.30 a.m. Wed Dec 2.**

6.2.3 $X \sim N(86, 25)$, $Z = (X - 86)/5 \ \sim \ N(0, 1)$.

(a) 10% of 86 is 8.6, and 8.6/5=1.72 SD. $P(Z \leq 1.72) \ = \ 0.9573$ (from table)

So $P(Z > 1.72) = 1 - 0.9573 = 0.0427$, and $P(X < 86 - 8.6) + P(X > 86 + 8.6) \ = \ 2 \times 0.0427 = 0.0854$.

(b) 75th percentile of $Z$ is 0.675 (from table), so of $X$ is $86 + 5 \times 0.675 \ = \ 89.37$.

(c) The 80 th percentile of $Z$ is 0.84 (from table), so 20th percentile is -0.84.

Hence for $X$ the 20th percentile is $86 - 5 \times 0.84 \ = \ 81.8$.

(d) The 95th. percentile of $Z$ is 1.645, so $Z$ lies between $\pm 1.645$ with probability 90%.

Hence for $X$, there is prob 90% that it lies within $86 \pm 5 \times 1.645 \ = \ 86 \pm 8.225$.

6.2.5 Bags have weight $X \ \sim \ N(5.13, 0.08^2)$. Let $Z \sim N(0, 1)$.

(a) $X = 5$ is $Z \ = \ (5 - 5.13)/.08 \ = \ -1.625$.

$P(Z < -1.625) \ = \ P(Z > 1.625) \ = \ 1 - \Phi(1.625) \ = \ 1 - 0.948 \ = \ 0.052$.

(b) 5.0 is to be the 1 percentile. The 99th percentile of $Z$ is 2.33, so the 1 percentile is -2.33.

Thus we need the mean of $X$ to be $5 + 2.33 \times 0.08 \ = \ 5.186$.

6.4.9 Let “success” be favor candidate B. We want $P(Bin(1500, 0.47) < 750)$, since then the poll will correctly predict B will lose. This Binomial has mean $1500 \times 0.47 \ = \ 705$ and variance 373.65, or $Z = (750 - 705)/\sqrt{373.65} \ = \ 2.33$, so the probability is 99%.

6.4.13 We compute the probability of the complement, that no one succeeds.

The probability of not succeeding is 1023/1024, so the probability noone succeeds is $(1023/1024)^{2000} \ = \ 0.1417$, and the probability at least 1 succeeds is 0.8583.

Using the Poisson approximation we have mean $2000/1024 \ = \ 1.953$, and the probability a Poisson with this mean is 0 is $exp(-1.953) \ = \ 0.1418$, so this would give 0.8582.

6.4.14 First we need the probability that a $Bin(600, 1/6)$ is at least 125. For this we use the Normal approximation, and we will use a continuity correction, so we need the probability a $N(100, 83.33)$ is at least 124.5, corresponding to $Z = (124.5 - 100)/\sqrt{83.33} \ = \ 2.68$, so the probability of success is 0.0037.

Now we need to approximate the number of sucesses in 300 Bernoulli trials. So here we use the Poisson approximation $Y \ \sim \ \mathcal{P}o(\mu)$ with $\mu = 300 \times 0.0037 \ = \ 1.11$. The probability of at least 2 winners is $1 - P(Y = 0) - P(Y = 1) \ = \ 1 - (1 + 1.11) \times \exp(-1.11) \ = \ 0.3046$.

6.Supp.8

(a) $n = 90$ trials, $p = 1/36$: Binomial probability is $\binom{90}{3} 35^{87}/36^{90} \ = \ 0.2171$.

(b) Poisson approximation: mean $90/36 = 2.5$. Probability of exactly 3 is $\exp(-2.5)(2.5)^3/3! \ = \ 0.2138$.

(c) Normal approximation: mean 2.5, variance 2.43, and we will find the probability it is between 2.5 and 3.5 (continuity correction), corresponding to $Z \ \sim N(0, 1)$ being between 0 and $(3.5 - 2.5)/\sqrt{2.43} \ = \ 0.6415$. So the required probability is $\Phi(0.6451) - \Phi(0) \ = \ 0.74 - 0.5 \ = \ 0.24$.

(d) Well the Poisson is surprisingly good (I think), as $n$ is not that large, and $p$ not that small. However, clearly the Normal will not do well –the count is down near 0, and the Binomial distribution quite skewed, and we are approximating the probability of a single discrete outcome, by a range that is a substantial chunk of the standardized Normal.