Homework 7; Due 11.30 a.m. Mon Nov 23.

4.2.16, (a) $\mathrm{E}(X+Y) = \mathrm{E}(X)+\mathrm{E}(Y) = 2+3=5$ Expectations always add.

(b) $\mathrm{E}(2XY) = 2\mathrm{E}(X)\mathrm{E}(Y) = 2.3.2 = 12$. Only because X,Y independent.

(c) $\mathrm{E}((3X-Y)^2) = \mathrm{E}(9X^2-6XY+Y^2) = 9\mathrm{E}(X^2)-6\mathrm{E}(X).\mathrm{E}(Y)+\mathrm{E}(Y^2) = 9.6-6.3.2+13 = 31$. The independence is used only for the $\mathrm{E}(XY)$ term.

(d) $[\mathrm{E}(3X-Y)]^2 = [3.2-3]^2 = 9$. Note the difference between (c) and (d) is $\mathrm{var}(3X-Y) = 31-9=22$.

1.Supp.11, $p = P(\text{good}) = 0.1+0.1-0.01 = 0.19$. Let G denote "good" and B denote "bad".

(a) $P(BBB) = (1-0.19)^3 = 0.5314$.

(b) $P(BBG) = (1-0.19)^2\times 0.19 = 0.1247$.

(c) $P(4G \text{ in first } 10) = P(Bin(10,0.19)=4) = \binom{10}{4}0.19^4 0.81^6 = 210\times .0013\times .2824 = .0773$

7.3.6, $p = P(\text{success}) = 1/3$.

(a) $P(\text{Fourth success is after 10 failures}) = P(\text{3 success in 13 tries; then success})$

$$= \binom{13}{3}(1/3)^3(2/3)^{10}(1/3) = 286\times 2^{10}/3^{14} = 0.0612.$$

(b) $P(\text{Third success on 8th try}) = P(\text{ 2 success in 7 tries; then success })$

$$= \binom{7}{2}(1/3)^2(2/3)^5(1/3) = 21\times 2^5/3^8 = 0.1024.$$

7.3.7 (a)(b)(c), $p = P(6) = 1/6$; success here is rolling a 6.

(a) Expected number of rolls to 6th 6 is 6 times expected tries to first 6, $= 6/(1/6) = 36$. So expected number of non-6 is $36-6 = 30$.

(b) Expected number of rolls we just did: 36

(c) $P(30F \text{ before } 6thS) = P(\text{ 30 } F \text{ in 35 tries, then S}) = \binom{35}{5}(1/6)^6(5/6)^{30} = 324632\times 5^{30}/6^{36} = 0.02931$.

6.4.4, $p=0.001$, $n=150$

Binomial probability that $Bin(150,0.001)$ is 0 is $.999^{150}=0.8606$, so probability at least 1 is $1-0.8606 = 0.1394$.

Approximate by Poisson with mean $\mu=np = 0.15$. Probability this r.v. takes value 0 is $\exp(-0.15)=0.8607$, so Poisson approx is $(1-0.8607) = 0.1393$.

7.4.3. $n=80$, $p=1/20$, the expected number of successes is 80/20 or 4.

We approximate by a Poisson mean $\mu=4$. The probability this r.v. takes value 4 is $\exp(-4)4^4/4! = 0.01831\times 32/3 = 0.1954$.