Homework 5; Due 11.30 a.m. Wed Nov 4.

1.S.7 $f_X(x) = Cx^2(1-x)^2$ on $0 < x < 1$ and 0 otherwise.
(a) $\int_0^1 x^2(1-x)^2\ dx \ = \ \int_0^1 (x^2 - 2x^3 + x^4)\ dx \ = \ (1/3) - (2/4) + (1/5) \ = \ 1/30$, so $C = 30$.
(b) $P(0 < X \leq z) \ = \ \int_0^z f_X(x)dx \ = \ 30(z^3/3 - z^4/2 + z^5/5)$ for $0 < z < 1$.
(c) $P(0.5 < X < 1) \ = \ 1 - P(0 < X < 0.5) \ = \ 1 - 30(1/24 - 1/32 + 1/160) \ = \ 1 - (5/4 - 15/16 + 3/16) \ = \ 1/2$
(Or, use symmetry.)

3.1.7 (a) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 \ = \ 55$,
so pmf is $P(X = x) \ = \ 1/55, 4/55, 9/55, 16/55$ and $25/55$, for $x = 1, 2, 3, 4, 5$ respectively.
(b) $P(X \in (1, 3]) \ = \ P(X = 2) + P(X = 3) \ = \ 13/55$.

3.1.8 (a) $\int_1^5 x^2 dx \ = \ [x^3/3]_1^5 \ = \ 124/3$,
so $f_X(x) \ = \ 3x^2/124$ if $1 < x < 5$ and 0 otherwise.
(b) $P(1 < X \leq 3) \ = \ \int_1^3 3x^2/124\ dx \ = \ (27 - 1)/124 \ = \ 13/62$. (Note this is smaller than 3.1.7 (b)).

3.2.3 (a) $F_X(1) = P(X \leq 1) = P(X = 0) + P(X = 1) = 4.2 \times e^{-3.2} \ = \ 0.1712$
(b) $F_X(0.99) = P(X \leq 0.99) = P(X = 0) \ = \ 0.0408$
(c) $F_X(3.2) = P(X \leq 3.2) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$
$= \ e^{-3.2}(1 + 3.2 + 5.12 + 5.46) \ = \ 0.602.$
(d) $F_X(-4.5) = 0$, since X cannot be negative.
(e) $\lim_{x \uparrow 2} F_X(x) \ = \ P(X < 2) \ = \ P(X = 0) + P(X = 1) \ = \ 0.1712$ as in (a).

3.2.7 (a) $\int_0^1 x^a\ dx \ = \ 1/(a+1)$ so $f_X(x) = (a+1)x^a$ on $0 < x < 1$ and 0 otherwise.
(b) $\int_0^z (a+1)x^a\ dx \ = \ z^a$, so $F_X(z) = 0$ for $z < 0$, $F_X(z) = z^{a+1}$ for $0 \leq z \leq 1$, and $F_X(z) = 1$ for $z > 1$.

3.3.7 (c),(d) only. The CDF is $F_X(x) = 1 - \exp(-2x)$ on $0 < x < \infty$.
Hence $P(X > 0.5) \ = \ \exp(-1) \ = \ 0.3496/0.3679 \ = \ 0.3679$ and $P(X \leq 2) \ = \ 1 - \exp(-4) \ = \ 0.9817$.
(c) $P(X \leq 2 \mid X > 0.5) \ = \ P(0.5 < X <\leq 2)/P(X > 0.5) \ = \ (0.9817 - (1 - 0.3679))/0.3679 \ = \ 0.9502$.
(d) $P(X > 0.5 \mid X \leq 2) \ = \ P(0.5 < X <\leq 2)/P(X \leq 2) \ = \ 0.3496/0.9817 \ = \ 0.3561$