

Homework 4; Due 11.30 a.m. Wed October 28.

1.2.4 (a,b,c); Let X be number of tails (T) before second head (H).

$$P(X = k) = P(k T \text{ before 2nd } H) = P(k T \text{ in } (k + 1), \text{ then } H)$$

$$= \binom{k+1}{1} (1/2)^{k+1} \cdot (1/2) = (k+1)(1/2)^{k+2}. \quad (k = 0, 1, 2, 3, \dots).$$

$$(a) P(X > 2) = 1 - P(X = 0) - P(X = 1) - P(X = 2) = 1 - 1/4 - 2 \cdot (1/8) - 3 \cdot (1/16) = 5/16.$$

$$(b) P(X > 3) = P(X > 2) - P(X = 3) = (5/16) - 4 \cdot (1/2)^5 = 3/16.$$

$$(c) P(X > 4) = P(X > 3) - P(X = 4) = (3/16) - 5 \cdot (1/2)^6 = 7/64.$$

1.2.5, Let X be number of tosses to get the first head.

$$(a) P(X = k) = P((k-1)T \text{ then } H) = (1/2)^k \text{ for } k = 1, 2, 3, 4, \dots$$

$$(b) P(X > 2) = 1 - P(X = 1) - P(X = 2) = 1 - 1/2 - 1/4 = 1/4 \text{ (or } P(TT)).$$

$$P(X > 3) = P(X > 2) - P(X = 3) = (1/4) - (1/8) = 1/8. \text{ (or } P(TTT)).$$

$$P(X > k) = P(\text{first } k \text{ are all } T) = (1/2)^k.$$

$$(c) P(X \text{ odd}) = P(X = 1) + P(X = 3) + P(X = 5) + \dots = (1/2) + (1/8) + (1/32) + \dots \\ = (1/2)(1 + (1/4) + (1/4)^2 + \dots) = (1/2)/(1 - (1/4)) = 2/3.$$

1.2.8 (a,b,c): $P(X = k) = \exp(-3) 3^k / k!$

$$(a) P(X = 0) = \exp(-3) = 0.0497$$

$$(b) P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = \exp(-3)(1 + 3 + 3^2/2) = 0.423.$$

$$(c) P(X = 3) = \exp(-3) \cdot 3^3 / 6 = 0.224.$$

1.2.10, $\Omega = \{1, 2, 3, 4, 5, 6\}$ with probabilities $1/91, 4/91, 9/91, 16/91, 25/91, 36/91$.

$$(a) P(X \text{ even}) = P(X = 2) + P(X = 4) + P(X = 6) = (4 + 16 + 36)/91 = 56/91 = 0.615.$$

$$(b) P(X \leq 2) = P(X = 1) + P(X = 2) = (1 + 4)/91 = 5/91 = 0.055.$$

$$(c) P(X \text{ square}) = P(X = 1) + P(X = 4) = (1 + 16)/91 = 17/91 = 0.187.$$

1.3.1(a,b,c), $P(a \leq X < b) = \exp(-0.37a) - \exp(-0.37b)$

$$(a) P(2 \leq X < 4) = \exp(-0.74) - \exp(-1.48) = 0.249.$$

$$(b) P(2 \leq X < \infty) = \exp(-0.74) = 0.477.$$

$$(c) P(0 \leq X < 1.5) = 1 - \exp(-0.555) = 0.426.$$

1.3.14: Area of disc radius r is πr^2 . Area of disc radius 1 is π .

$$(a) P(X < 1/3) = (\pi/9)/\pi = 1/9.$$

$$(b) P(1/3 < X < 2/3) = P(X < 2/3) - P(X < 1/3) = ((4\pi/9) - (\pi/9))/\pi = (4/9 - 1/9) = 1/3.$$

$$(c) P(X > 2/3) = 1 - P(X < 2/3) = 1 - (4/9) = 5/9.$$

$$(d) P(X \leq r) = \pi r^2 / \pi = r^2.$$