Homework 3; Due 11.30 a.m. Wed October 21.

2.2.7 and 2.4.7:

$P(B) = P(B|A) = P(B\cap A)/P(A)$ So $P(A|B) = P(B\cap A)/P(B) = P(B)P(A)/P(B) = P(A)$.
$A\subset B$, A, B indep: $P(A) = P(A\cap B) = P(A)P(B)$ So $P(A)=0$ or $P(B)=1$.

2.2.13: $P(A\cap B) = P(A)+P(B)-P(A\cup B) = 0.68+0.53-0.75 = 0.46$.
So $P(B \mid A) = P(A\cap B)/P(A) = 0.46/0/68 = 0.676$.

2.2.14: (a) $P(B_1\cap W_2\cap B_3) = (3/10).(7/11).(4/12) = 7/110$ (same as Ex. 2.2.12)
(b) $P(W_1\cap B_2\cap B_3) = (7/10).(3/11).(4/12) = 7/110$ again.
(c) $P(B_2) = P(B_1\cap B_2)+P(W_1\cap B_2) = (3/10).(4/11)+(7/10).(3/11) = 33/110 = 3/10 = P(B_1)$.

2.3.1: (a) Let G be thinks good job, D Democrat, R republican, I independent.
By law of total probability:

$$P(G) = P(G|D)P(D)+P(G|R)P(R)+P(G|I)P(I) = .8\times .35+.45\times .4+.55\times .25 = .598$$

(b) By Bayes' formula: $P(D \mid G) = P(G|D)P(D)/P(G) = .8\times .35/0.598 = 0.469$.

2.3.8: Let B_i be event ith ball is blue. $P(B_1) = 5/10 = 1/2$.
$P(B_2) = P(B_2|B_1)P(B_1) + P(B_2 \mid B_1^c)P(B_1^c) = (6/11)\times(1/2) + (5/11)\times(1/2) = 1/2$
From Bayes' formula: $P(B_1 \mid B_2) = P(B_2|B_1)P(B_1)/P(B_2) = 6/11 = P(B_2|B_1)$.
That is, the two conditionals **are** the same **in this case** because $P(B_1) = P(B_2)$.

2.4.1: $\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ each prob 1/8.
(a) $P(A) = P(\text{heads on 1 st}) = 1/2$, $P(B) = P(\text{even heads}) = 1/2$, $P(A\cap B) = P(HHT\cup HTH) = 1/4$.
So yes, A and B are independent.
(b) $P(A) = P(\text{no head in 1,2}) = 1/4$, $P(B) = P(\text{no head in 2,3}) = 1/4$, $P(A\cap B) = P(TTT) = 1/8$.
So A and B are not independent.
(c) $P(A) = P(\text{even in 1,2}) = P(HHH, HHT, TTH, TTT) = 1/2$,
$P(B) = P(\text{even in 2,3}) = P(HHH, THH, HTT, TTT) = 1/2$,
$P(A\cap B) = P(HHH\cup TTT) = 1/4$. So yes, A and B are independent.