Homework 2; Due 11.30 a.m. Wed October 14.

2.1.5 (a) P(defective and made by machine 2) $= 0.3 \times .04 = 0.012$.

P(defective and made by machine 3) $= 0.2 \times .02 = 0.004$.

P(defective and made by machine 4) $= 0.15 \times .02 = 0.003$.

(b) P(defective) = sum of P(defective and made by machine i) $= 0.0175 + 0.012 + 0.004 + 0.003 = 0.0365$

(c) P(not defective) = 1−P(defective) $= 0.9635$.

(d) P(made by machine 2 and not defective) $= 0.3 \times (1 - 0.04) = 0.288$.

2.1.12 Total number of choices is $\binom{52}{3}$. You do **not** win if you get no face cards. The number of ways of doing this is $\binom{40}{3} \times \binom{12}{0}$, so the probability of winning is $1 - \binom{40}{3}/\binom{52}{3} = 0.5529$.

2.1.15 (a) The number of ways of chosing 4 from 8 is $\binom{8}{4}$.

The number of ways of choosing 4 from 6 (G_3, G_4, G_5, G_6, G_7, G_8) is $\binom{6}{4}$.

So P(neither G_1 nor G_2 is chosen) $= \binom{6}{4}/\binom{8}{4} = (6!\ 4!\ 4!)/(4!\ 2!\ 8!) = (4 \times 3)/(8 \times 7) = 3/14$.

(b) We want the probability that G_1 and G_2 are not chosen, but G_3 is chosen.

It is easier to consider the probability that none of G_1, G_2 and G_3 are all not chosen. There are $\binom{5}{4}$ ways of choosing 4 from $G_4,\ G_5,\ G_6,\ G_7,\ G_8$, so $P(G1, G2, G3 \text{ all not chosen}) = \binom{5}{4}/\binom{8}{4} = 5/70 = 1/14$.

So $P(G_1,\ G_2$ not chosen, but G_3 is chosen$) = 3/14 - 1/14 = 2/14 = 1/7$.

(Note: $P(not\ G_1, not\ G_2, not\ G_3) + P(not\ G_1, not\ G_2, yes\ G_3) = P(not\ G_1, not\ G_2)$ (why?))

2.1.16 (a) There are $n = 6^3 = 216$ possible outcomes. Of these $m = 6 \times 5 \times 4 = 120$ give rise to the event that the three are different (why?). So the probability is $120/216 = 20/36$.

(b) Here $m = 6 \times 1 \times 1 = 6$ give rise to the event that all three are the same (why?).

So the probability is $6/216 = 1/36$.

2.2.1 (a) $P(HTT \mid HTT \cup THT \cup TTH) = (1/2)^3/3(1/2)^3 = 1/3$

Note that this would be the same if the coin was unfair!

(b) $P(HTT \mid HTT \cup HTH \cup HHT \cup HHH) = (1/2)^3/4(1/2)^3 = 1/4$

This would **not** be the same if the coin were unfair!

(c) $P(HHH \mid HHH \cup HHT \cup HTH \cup THH) = (1/2)^3/4(1/2)^3 = 1/4$.

(d) $P(HHH \mid HHH \cup HHT) = (1/2)^3/2(1/2)^3 = 1/2$

2.2.3 (a) 1 is chosen, if chosen first or second: probability $1/100 + (99/100).(1/99) = 2/100 = 1/50$.

(b) Easiest way is to just eliminate 2, so we start off with 99 numbers, and probability is $1/99 + (98/99)(1.98) = 2/99$.

(c) Probability neither 1 nor 2 is first is 98/100, then choosing 1 second is 1/99.

Overall conditional probability is $(98/100) \times (1/99)/(98/100) = 1/99$

(d) Probability 2 is not first is (99/100). For 1 to be second, it also can not be first, so there are 98 choices for first, and then 1/99 chance that 1 is second.

Overall conditional probability is $(98/100) \times (1/99)/(99/100) = 98/99^2 = 98/9801$.