

1.

German (G)	English (E)		
	Yes	No	
Yes			0.3
No		0.5	0.7
	0.4	0.6	1.0

(a)  $P(E \cup G) = 1 - P(E^c \cap G^c) = 0.5.$

(b)  $P(E \cap G) = P(E) + P(G) - P(E \cup G)$   
 $0.3 + 0.4 - 0.5 = 0.2.$

(c) No  $E$  and  $G$  are not independent because  
 $P(E) \times P(G) = 0.12$  but  $P(E \cap G) = 0.2.$

2. (a) The number of ways of choosing 5 out of 8 is  $\binom{8}{5} = 8 \times 7 \times 6 / (3 \times 2 \times 1) = 56.$

(b) If Joe invites neither Fred nor Anna he must choose 5 out of the other 6. Then there are just  $\binom{6}{5} = 6$  choices.

(c) If Joe invites both Fred and Anna, then he must choose another 3 out of the other 6, so the number of choices is  $\binom{6}{3} = 6 \times 5 \times 4 / (3 \times 2 \times 1) = 20.$

3. (a) In a population 25% of people are type  $bb$ , 50% are type  $bg$ , and the remaining 25% have grey eyes. Let  $B_0$  be the event Sarah has brown eyes,  $bb$  is event Sarah is type  $bb$ , and  $bg$  is event Sarah is type  $bg$ .

$$P(bb | B_0) = 0.25 / (0.25 + 0.5) = 1/3$$

(b) Sarah marries Paul, who has grey eyes. Their first child has brown eyes: event  $B_1$ .

$$P(bb \cap B_0 \cap B_1) = P(B_1 | bb \cap B_0)P(bb \cap B_0) = 1 \times P(bb) = P(bb) = 0.25$$

$$P(bg \cap B_0 \cap B_1) = P(B_1 | bg \cap B_0)P(bg \cap B_0) = 0.5 \times P(bg) = 0.5 \times 0.5 = 0.25.$$

So  $P(bb | B_0 \cap B_1) = 0.25 / (0.25 + 0.25) = 1/2.$

(c) Sarah and Paul's second child also has brown eyes: event  $B_2$ .

$$P(bb \cap B_0 \cap B_1 \cap B_2) = P(B_1 \cap B_2 | bb \cap B_0)P(bb \cap B_0) = 1 \times 1 \times P(bb) = P(bb) = 0.25$$

$$P(bg \cap B_0 \cap B_1 \cap B_2) = P(B_1 \cap B_2 | bg \cap B_0)P(bg \cap B_0) = 0.5 \times 0.5 \times P(bg) = 0.5 \times 0.5 \times 0.5 = 0.125.$$

$$\text{So } P(bb | B_0 \cap B_1 \cap B_2) = 0.25 / (0.25 + 0.125) = 2/3.$$

(d) Sarah and Paul's third child has grey eyes; event  $G_3$ .

Sarah can no longer be of type  $bb$ : she must be  $bg$ .

$$P(bb | B_0 \cap B_1 \cap B_2 \cap G_3) = 0.$$

	each event			combined events			updated prob	
	bb	bg	grey	bb	bg	grey	bb	bg
Popn	0.25	0.5	0.25					
$B_0$	1	1	0	0.25	0.5	0	1/3	2/3
$B_1$	1	0.5	—	0.25	0.25	—	1/2	1/2
$B_2$	1	0.5	—	0.25	0.125	—	2/3	1/3
$G_3$	0	0.5	—	0	0.0625	—	0	1