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1. Permutations and combinations

There are $n! = \prod_{i=1}^{n} i = 1.2.3.4.\ldots.n$ permutations of n objects.

There are $\binom{n}{k} = n!/(k!(n-k)!)$ different combinations of k objects chosen from n.

2. Joint and conditional probabilities

If C and D are any events: $P(C \bigcup D) = P(C) + P(D) - P(C \bigcap D)$.

The conditional probability of C given D is $P(C \mid D) = P(C \bigcap D) / P(D)$.

C and D are independent if $P(C \bigcap D) = P(C).P(D)$.

3. Laws and theorems

Suppose $E_1, \ldots, E_k$ is a partition of Ω. That is $E_i \bigcap E_j$ is empty for all i,j, and $E_1 \bigcup E_2 \bigcup \ldots \bigcup E_k = \Omega$.

The law of total probability states that: $P(D) = \sum_{j=1}^{k} P(D \bigcap E_j) = \sum_{j=1}^{k} P(D \mid E_j)\ P(E_j)$

Bayes' Theorem states that: $P(E_i \mid D) = P(D \mid E_i)\ P(E_i)/P(D)$

Justify your answers briefly.

1. (*9 points total; 3 points each part*)

In Denmark, 40% of the adults can speak English and 30% of the adults can speak German, but 50% can **not** speak either English or German. A person from the adult population of Denmark is chosen at random.

(a) What is the probability this person can speak either English or German?

(b) What is the probability this person can speak both English and German?

(c) Is the event that this person can speak German independent of the event that this person can speak English?

2. (*9 points total; 3 points each part*)

(a) Joe has 8 friends he would like to take on a day tour, but his car will only hold 5 (in addition to himself).

How many different choices of 5 of his 8 friends are there?

(b) Joe learns that 2 of his 8 friends, Fred and Anna, are a couple, so he must invite either both or neither.

How many different choices are there if he invites neither Fred nor Anna?

(c) How many different choices are there if he invites both Fred and Anna?

3. (*16 points: 4 points each part*)

A very simplified version of the way eye-color is determined in humans is as follows. Each person has dark brown eyes (B) or light grey eyes (G). The people with brown eyes are of two genetic types, bb or bg. If a bb person marries someone with grey eyes, all their children will have brown eyes. If a bg person marries someone with grey eyes, each of their children will have brown eyes with probability 1/2 and grey eyes with probability 1/2, independently of the types of any previous children.

(a) In a population 25% of people are type bb, 50% are type bg, and the remaining 25% have grey eyes.

Let B_0 be the event Sarah has brown eyes, bb is event Sarah is type bb, and bg is event Sarah is type bg.

Given that Sarah does have brown eyes. what is the probability that Sarah is of type bb: that is, $P(bb \mid B_0)$?

(b) Sarah marries Paul, who has grey eyes. Their first child has brown eyes: event B_1.

Show $P(bb \cap B_0 \cap B_1) = 0.25$ and $P(bg \cap B_0 \cap B_1) = 0.25$.

What is the updated probability that Sarah is bb: $P(bb \mid B_0 \cap B_1)$?

(c) Sarah and Paul's second child also has brown eyes: event B_2.

Show $P(bb \cap B_0 \cap B_1 \cap B_2) = 0.25$ and $P(bg \cap B_0 \cap B_1 \cap B_2) = 0.125$.

What is the updated probability that Sarah is bb: $P(bb \mid B_0 \cap B_1 \cap B_2)$?

(d) Sarah and Paul's third child has grey eyes; event G_3.

What is the updated probability that Sarah is bb: $P(bb \mid B_0 \cap B_1 \cap B_2 \cap G_3)$?