You need NOT compute final numerical answers: a numerical formula is sufficient. However, you must explain/justify your answers.

1. *(12 points: 3 each part)*
A continuous random variable X has probability density function (pdf)

$$\begin{aligned} f_X(x) &= \tfrac{1}{2}(x+1) \quad \text{if } -1 < x < 1, \\ &= 0 \qquad\qquad\qquad \text{otherwise.} \end{aligned}$$

(a) Find the cumulative distribution function $F_X(x) \;=\; P(X \le x)$.
(Hint: $\int \frac{1}{2}(y+1)dy \;=\; (y+1)^2/4$.)

(b) Find $P(X \;>\; 0)$.

(c) Find $P(X^2 \;<\; 1/4)$.

(d) Find $P(\exp(X) \;\le\; 1)$.

2. *(12 points: 4 each part.)* Coffee mugs for a student union promotion are made in two colors, either blue or white. Since the white mugs are more popular, the manufacturer produces twice as many white mugs as blue. They are packed into crates randomly, so that each mug has probability 2/3 of being white and 1/3 of being blue, independently of other mugs in the crate.
The students decide to unpack mugs for display until they have at least 8 white mugs and at least 4 blue mugs.

(a) What is the probability the students unpack only 12 mugs in total?

(b) What is the probability the students unpack 8 white mugs and 8 blue mugs (16 in total)?

(c) Suppose the students do unpack 16 mugs, 8 of each color. An assistant randomly puts 8 mugs on each of two display shelves. What is the probability there are 4 white mugs and 4 blue mugs on each shelf?

3. *(12 points: 3 each part.)*
X, Y and Z are three random variables defined on the same sample space Ω.
It is known that $\mathrm{E}(X) = \mathrm{var}(X) = 1$, $\mathrm{E}(Y) = \mathrm{var}(Y) = 2$ and $\mathrm{E}(Z) = \mathrm{var}(Z) = 3$.
Additionally, the two random variables X and Y are independent.

(a) Find $\mathrm{E}(3X - 2Y + Z + 7)$.

(b) Find $\mathrm{var}(3Z + 17)$.

(c) Find $\mathrm{var}(3X - 2Y + 7)$.

(d) Find $\mathrm{E}(X(Y+2) \;+\; Z^2)$.