Justify your answers briefly.

1. (*12 points total; 3 points each part*)

In a student cafeteria, Burgers can be ordered with a variety of extras. In fact, 40% of the Burger orders include cheese, and 30% include bacon, but 40% of the orders are for a Burger with neither cheese nor bacon.

	Cheese (C)		
	Yes	No	
Bacon (B) Yes	0.1	0.2	**0.3**
No		**0.4**	
	0.4	0.6	

(a) $P(C^c) = 1 - 0.4 = 0.6$, $P(B \cap C^c) = 0.6 - 0.4 = 0.2$,
$P(B \cap C) = 0.3 - 0.2 = 0.1$

(b)No, not independent:
$0.1 = P(B \cap C) \neq P(B).P(C) = 0.3 \times 0.4 = 0.12$

(c) $P(B|C) = P(B \cap C)/P(C) = 0.1/0.4 = 0.25$

(d) $P(C|B) = P(B \cap C)/P(B) = 0.1/0.3 = 0.33$

2. (*8 points total; 4 points each part*)

There are 8 balls in an urn: 2 are blue, 2 are white, and 4 are red.

Three balls are to be selected at random from the urn.

(a) With replacement: on each draw $P(W) = P(B) = 2/8 = 1/4$ and $P(R) = 1/2$.

Probability all three draws are same color is

$$P(RRR) + P(WWW) + P(BBB) \;=\; (\tfrac{1}{2})^3 + (\tfrac{1}{4})^3 + (\tfrac{1}{4})^3 \;=\; (1/8) + (1/32) \;=\; 5/32.$$

(b) Without replacement, the only way all three can be the same color is if all three are red.

$$P(RRR) \;=\; (4/8) \times (3/7) \times (2/6) \;=\; 1/14.$$

3. (*16 points: 4 points each part*)

In a certain large population, families have 1, 2, or 3 kids with probabilities 3/16, 1/4 and 9/16, respectively. Note that a kid chosen randomly from a family size 1, 2, 3 is the youngest in their family with probabilities 1, 1/2 and 1/3, respectively. A random family is selected for a special sales promotion.

(a) By law of total probablity, probability kid is youngest is

$$1 \times (3/16) \;+\; (1/4) \times (1/2) \;+\; (1/3) \times (9/16) \;=\; (3 + 2 + 3)/16 \;=\; 1/2.$$

(b) Conditional probabilities for eldest are 1, 1/2 and 1/3, just as for youngest. So this probability is also (1/2).

(A few read this as probability kid is eldest **given** kid in (a) is youngest – this is harder, answer 3/8, but if done right I accepted it.)

(c) By Bayes theorem, probabilities are

$1 \times (3/16)/(1/2)$, $(1/2) \times (1/4)/(1/2)$ and $(1/3) \times (9/16)/(1/2)$ or 3/8, 1/4 and 3/8.

(d) A random kid is chosen from **the same** family. (This choice is "with replacement": the same kid could be chosen again.) Again the kid chosen is the youngest in his/her family.

Using Bayes theorem again with updated probabilities: the numerators of the updated probabilities are $1 \times (3/8)$, $(1/2) \times (1/4)$ and $(1/3) \times (3/8)$, and sum of these is $(3/8) + (1/8) + (1/8) \;=\; 5/8$, for updated probabilties of $1 \times (3/8)/(5/8)$, $(1/2) \times (1/4)/(5/8)$ and $(1/3) \times (3/8)/(5/8)$, or 3/5, 1/5 and 1/5.

(Note if we do all details of the computations $P(Y_2|Y_1) = 5/8$, $P(Y_1 \cap Y_2) = 5/16$ – be careful.)