**STAT 340: FALL 2009: Information sheet for Final Exam: 2:30-4:20, Wed. Dec. 16**

(A Normal probability table is on the back of this sheet.)

## 1. Permutations and combinations

There are $n! = \prod_{i=1}^{n} i = 1.2.3.4.\ldots.n$ permutations of $n$ objects.

There are $\binom{n}{k} = n!/(k!(n-k)!)$ ways of choosing $k$ objects from $n$.

## 2. Joint and conditional probabilities

If $C$ and $D$ are any events: $P(C \bigcup D) = P(C) + P(D) - P(C \bigcap D)$.

The conditional probability of $C$ given $D$ is $P(C \mid D) = P(C \bigcap D) \,/\, P(D)$.

$C$ and $D$ are independent if $P(C \bigcap D) = P(C).P(D)$.

## 3. Laws and theorems

Suppose $E_1, \ldots, E_k$ is a partition of $\Omega$. That is $E_i \bigcap E_j$ is empty, and $E_1 \bigcup E_2 \bigcup \ldots \bigcup E_k = \Omega$.

The law of total probability states that: $P(D) = \sum_{j=1}^{k} P(D \bigcap E_j) = \sum_{j=1}^{k} P(D \mid E_j)\, P(E_j)$

Bayes' Theorem states that: $P(E_i \mid D) = P(D \mid E_i)\, P(E_i)/P(D)$

## 4. Random variables and distributions

| **4. Random variables and distributions** | **discrete (mass)** | **continuous (density)** |
|---|---|---|
| Probability mass/density function | pmf: $P(X = x) = p_X(x)$ | pdf: $f_X(x)$ |
| defined for | all $x$ with $p_X(x) > 0$ | $-\infty < x < \infty$ |
| Cumulative distrib. func. cdf: $P(X \le x)$ | $F_X(x) = \sum_{y \le x} p_X(y)$ | $F_X(x) = \int_{-\infty}^{x} f_X(y) dy$ |
| Expectation $\mathrm{E}(X)$ | $\sum_x x p_X(x)$ | $\int_{-\infty}^{\infty} x f_X(x) dx$ |
| | **provided** the sum/integral converges. | |
| Result: $\mathrm{E}(g(X))$ | $\sum_x g(x) p_X(x)$ | $\int_{-\infty}^{\infty} g(x) f_X(x) dx$ |

Variance: $\mathrm{E}((X - (\mathrm{E}(X)))^2) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

## 5. Standard distributions

| **5. Standard distributions:** $p_X$ or $f_X$ | values | mean $\mathrm{E}(X)$ | variance |
|---|---|---|---|
| (a) Binomial; $B(n,p)$; index $n$, parameter $p$; | | | |
| $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ | $k = 0, 1, 2, ..., n$ | $np$ | $np(1-p)$ |
| (b) Geometric; $G(p)$; parameter $p$; | | | |
| $P(X = k) = p(1-p)^{k-1}$ | $k = 1, 2, 3, 4, ......$ | $1/p$ | $(1-p)/p^2$ |
| (c) Negative Binomial; $NegB(r,p)$ | | | |
| $P(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$ | $k = r, r+1, r+2, ...$ | $r/p$ | $r(1-p)/p^2$ |
| (d) Poisson; $Po(\mu)$; parameter $\mu$; | | | |
| $P(X = k) = \exp(-\mu)\mu^k/k!$ | $k = 0, 1, 2, 3, .....$ | $\mu$ | $\mu$ |
| (e) Hypergeometric $(N, m, n)$; | | | |
| $P(X = k) = \binom{m}{k}\binom{N-m}{n-k}/\binom{N}{n}$ | $k = 0, ..., n$<br>$k \ge m + n - N,\ k \le m$ | $nm/N$ | — |
| (f) Normal, $N(\mu, \sigma^2)$; | | | |
| $f_X(x) = (1/\sqrt{2\pi\sigma^2}) \exp(-(x-\mu)^2/2\sigma^2)$ | $-\infty < x < \infty$ | $\mu$ | $\sigma^2$ |
| (g) Uniform on interval $(a,b)$; $U(a,b)$; | | | |
| $f_X(x) = 1/(b-a)$ | $a < x < b$ | $(a+b)/2$ | $(b-a)^2/12$ |
| (h) Exponential, $E(\lambda)$, rate parameter $\lambda$; | | | |
| $f_X(x) = \lambda \exp(-\lambda x)$ | $0 \le x < \infty$ | $1/\lambda$ | $1/\lambda^2$ |

**6. Summation of series:**

(a) $\sum_{i=0}^{n-1} x^i \;=\; (1 + x + x^2 + ... + x^{n-1}) \;=\; (1 - x^n)/(1 - x)$

(b) $\sum_{i=0}^{\infty} x^i/i! \;=\; (1 + x + x^2/2 + x^3/6 + ...) \;=\; \exp(x)$

**7. Here is a copy of the Normal distribution probability table from Kelly P.601**

Tabulated is the cdf $P(X \leq x)$ for a random variable $X$ which is Normal with mean 0 and variance 1.